1372

RUDIMENTARY TREATISE

ON

MENSURATION,

ARRANGED

FOR THE USE OF SCHOOLS AND PRACTICAL MEN:

COMPREHENDING THE ELEMENTS OF

MODERN ENGINEERING.

Fourth Edition, with Additions and Corrections.

By T. BAKER, C.E.,

Author of "Theodolite Surveying, Levelling, &c." in the Ninth Edition of Nesbit's Surveying; "Railway Engineering;" "Land and Engineering Surveying;" "Statics and Dynamics, Elements of Mechanism;" "Integration of Differentials;" &c., &c.

LONDON:

JOHN WEALE, 59, HIGH HOLBORN.

1859.

NOTICE BY THE PUBLISHER.

In issuing a Fourth Edition of the present work, I feel it incumbent upon me, as the Publisher, to offer a few remarks in reference to an article in the *Athenæum* of March 13th, 1858, which article, under the assumed character of a Review of Baker's Mensuration, contains charges injurious alike to the Author and to myself, and to give publicity to which seems to have been the sole object of the writer.

The Reviewer has coupled together, in his production, two books on Mensuration—one by Mr. Elliot, and the other by Mr. Baker; and the single point he aims at establishing, is that the latter Author has plagiarised from the former.

Upon his own showing, the Reviewer has been in previous correspondence with Mr. Elliot, and has been supplied with letters, passing between Mr. Elliot, Mr. Baker, and myself; by aid of these he is enabled, he thinks, to substantiate a charge of plagiarism against Mr. Baker, and consequently to implicate me in dealing in stolen goods.

Now, to those who have not read the Reviewer's remarks, it is necessary to state that all the alleged plagiarisms consist of certain schoolboy *questions*, with which books on Mensuration usually abound, and which are given as exercises for schoolboys to work out. As everybody knows, these questions involve in general but arbitrary suppositions, and seldom have any reference to actually existing circumstances or matters of fact. I here quote two, which—although the *numbers are altered* in Mr. Baker's statements of them—are adduced as plagiarisms from Mr. Elliot:

1. "A ladder is to be placed so as to reach a window, the sill of which is $67\frac{1}{2}$ feet from the ground; the foot of the ladder cannot be brought nearer than 36 feet from the wall: what length of ladder will be sufficient? Answer, $76\frac{1}{2}$ feet."

In Mr. Baker's version, the given members being different, the answer is $38\frac{1}{4}$ feet.

2. "A ladder standing upright beside a wall 100 feet high just reaches the top; how far may the foot of the ladder be removed from the wall, and still reach within 6 inches of the top? Answer, 10 feet."

This question too, though substantially the same in Baker (see question 11, p. 21), is nevertheless considerably modified, and is certainly not copied.

Now, I would ask any candid person, familiar with schoolbooks of this kind, whether questions such as the above are not common to them *all; and* whether there is a particle of merit in framing them or *a particle of* demerit in copying them?

If the Reviewer wish to further employ his talents in this kind of criticism, I can furnish him with an ample crop of materials. Or, if he have ever written a book on Mensuration or on Algebra himself, I will undertake to supply him with the like materials from his own work, so that he may profitably review himself.

Did the Reviewer ever see the question about the post being so much in mud, so much in water, and so much above the water?

I dare say he can tell me in what book on Algebra that occurs: can he tell in what book on Algebra it does *not* occur?

I might make the same inquiry about the *fish*, which every book on Algebra serves up—head, body, and tail. And he ought to know that questions about ladders strutting across streets, and reaching windows, are quite as hackneyed as these.

Such is the nature of the literary delinquencies brought against the author of this book: but the Reviewer gathers—not from the book—but from the *private letters* before alluded to, that Mr. Baker has affirmed what is not true. Mr. B. denies having seen Mr. Elliot's work, and the Reviewer dwells a good deal upon his moral delinquency. Is it the office of the Reviewer to assume the function of a public corrector of morals, even though the offence be confined to a private letter?

But, even on this point, the Reviewer's evident desire to damage the Author and his work, has hurried him to hasty conclusions. Mr. Elliot has written *two* works on Mensuration, in size and appearance totally unlike, and put forth by different publishers.

Mr. Baker, as was his duty, collected all the books on the subject he could, during the *compilation* of his own; and he did so expressly with the view of *selecting questions:* I presume the general practice of all compilers of such treatises. He has no doubt about having seen *one* of Mr. Elliot's two works, but the very different book from which he is charged with copying, I am persuaded he never did see. As the Reviewer is so rigid a judge in matters of plagiarism, I think he ought, in consistency, to rebuke Mr. Elliot for copying even from himself.

I have only to say, in conclusion, that it has always been my most anxious endeavour to secure, for my series of Rudimentary Treatises, writers of ability and reputation; and I respectfully, but confidently, leave the public to judge whether this endeavour has not been on the whole successful. And I cannot but complain that any writer in a literary journal should attempt to frustrate my honourable efforts, and injure property on which I have expended a very large outlay, by insinuations, as unjust as they are injurious, that I connive at an infringement of the rights of others.

JOHN WEALE.

INTRODUCTION.

r will at once be seen that condensation of the materials prouced by previous authors, and the introduction of a judicious election of matter, adapted to the expanded intellect of the resent age, are the proper requisites for a work on Mensuration. 'o this plan, the author trusts, from his long experience in enineering pursuits, that he has strictly adhered. In the first art, on PRACTICAL GEOMETRY, numerous examples are introuced, wherein the dimensions of certain parts are given to find he dimension of their corresponding parts, which has been rarely r never done by previous authors. This part is succeeded by a econd part, on the MENSURATION OF LINES; which is not dded for the sake of novelty only, but because it seemed to be he natural order of a work of this kind. The third and fourth Parts treat of the MENSURATION OF SUPERFICIES AND OF SOLIDS; while in all the three last-named Parts the rules are not only given in words at length, in the usual way, but the same rules are expressed by FORMULÆ, together with other formulæ depending thereon, by which the rules receive considerable extension. Some of the rules and examples are taken *verbatim* from *Dr. Hutton's Mensuration;* for the author conceives that it would be disreputable to attempt, by *verbal alterations* in such rules, to give an air of originality to his work, as all other authors have done since Dr. H.'s time: the originality of this work consists in the new matter, everywhere added, to adapt it to the wants of modern times. Timber measuring and Artificers' work, the latter with considerable modern improvements, are next introduced, with concise and practical methods of finding the surfaces and solidities of vaulted roofs, arches, domes, &c.

Concise, and the author trusts, clear systems of land surveying, levelling, laying out railway curves and finding the contents of railway cuttings, complete the work, and serve as an introduction to the author's *Land and Engineering Surveying,* which contains everything adapted to modern practice that can be desired, an extension to this subject having been first given by the author, not found in any work previous to those written by him, see pages 179 and 203 of that work.

The demonstrations of all the rules and formulæ, in the four

leading parts of the work, will be found in *Dr. Hutton's Large Mensuration* and in the *Rudimentary Geometry;* the remainder of the demonstrations will be found in the author's *Railway Engineering* or in his *Land and Engineering Surveying.*

Conic Sections and their solids are very briefly treated of in the four preceding parts of the work, and chiefly in as far as they may be useful to those who may intend to become excise officers, whose actual practice is best learnt from an experienced officer. Thus an extended article, such as is usually given by other authors, is avoided, as not being generally useful to practical men.

The weights and dimensions of balls and shells may be found by Prob. VIII., Part IV., in conjunction with the Table and Rules for finding the specific gravities of bodies, if required.

The method of piling balls and shells, finding their number in a given pile, and the quantity of powder contained in a given shell or box, form no essential part of a work on mensuration, being only useful in an arsenal, and are, therefore, also omitted. The author has thus secured ample space for the discussion of subjects really useful to the great majority of students and practical men, in the compass of a volume less than half the size and one-fifth of the price of the works of his predecessors ; beside adding matter, adapted to the wants of modern times, not foun in any existing work on mensuration.

The plan being thus briefly detailed, it will now be proper, previous to studying the following work, to give the following

DIRECTIONS FOR BEGINNERS.

The beginner, for a first course, may omit the Problems be yond the thirty-second in Practical Geometry, and Problem III., VIII., IX., XI., and XII., in the Mensuration of Li with the formulæ and examples depending on them. He may also omit all the formulæ in the Mensuration of Superficies and Solids, with the examples depending on them, as well as the Problems beyond the tenth in the Mensuration of Solids, except it is required he should learn the method of gauging casks, in which case omit only the two last problems. But if he require an extensive knowledge of some or all the subjects here treated of, he will do well to learn the use of such of the formulæ and the other parts, omitted in the first course, according to what he may require as a practical man.

CONTENTS.

PART I.

PART II.

PART III.

PART IV.

Mensuration

MENSURATION.

MENSURATION treats of the various methods of measuring and estimating the dimensions and magnitudes of figures and bodies. It is divided into four parts, viz., Practical Geometry, and Mensuration of Lines, of Superfices, and of Solids, with their several applications to practical purposes.

PART I.

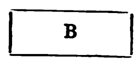
PRACTICAL GEOMETRY.

DEFINITIONS.

1. *A point* has no dimensions, neither length, breadth, nor thickness.

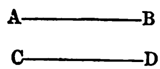
2. *A line* has length only, as A.

3. *A surface or plane* has length and breadth, as B.

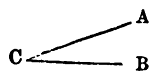


4. *A right or straight line* lies wholly in the same direction, as A B.

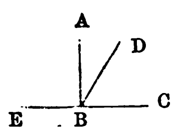
5. *Parallel lines* are always at the same distance, and never meet when prolonged, as A B and C D.



6. *An angle* is formed by the meeting of two lines, as A C, C B. It is called the angle A C B, the letter at the angular point C being read in the middle.



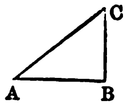
7. *A right angle* is formed by one right line standing erect or perpendicular to another; thus, A B C is a right angle, as is also A B E.



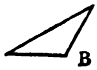
8. *An acute angle* is less than a right angle, as D B C.

9. *An obtuse angle* is greater than a right angle, as D B E.

10. *A plane triangle* is a space included by three right lines, and has three angles.

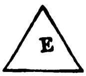


11. *A right angled triangle* has one right angle, as A B C. The side A C, opposite the right angle, is called the hypothenuse; the sides A B and B C are respectively called the base and perpendicular.

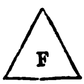


12. *An obtuse angled triangle* has one obtuse angle, as the angle at B.

13. *An acute angled triangle* has all its three angles acute, as D.

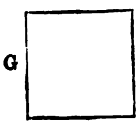


14. *An equilateral triangle* has three equal sides, and three equal angles, as E.



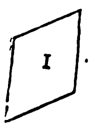
15. *An isosceles triangle* has two equal sides, and the third side greater or less than each of the equal sides as F.

16. *A quadrilateral figure* is a space bounded by four right lines, and has four angles; when its opposite sides are equal, it is called a *parallelogram.*

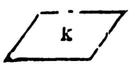


17. *A square* has all its sides equal, and all its angles right angles, as G.

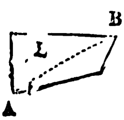
18. *A rectangle* is a right angled parallelogram, whose length exceeds its breadth, as B, (see figure to definition 2).



19. *A rhombus* is a parallelogram having all its sides and each pair of its opposite angles equal, as I.



20. *A rhomboid* is a parallelogram having its opposite sides and angles equal, as K.



21. *A trapezium* is bounded by four straight lines, no two of which are parallel to each other, as L. A line connecting any two of its angles are called the *diagonal,* as A B.

PROBLEMS IN PRACTICAL GEOMETRY.

(In solving the five following problems only a pair of common compasses and a straight edge are required; the problems beyond the fifth require a scale of equal parts; and the two last a line of chords: all of which will be found in a common case of instruments.)

Problem I.

To divide a given straight line A B *into two equal parts.*

From the centres A and B, with any radius, or opening of the compasses, greater than half A B, describe two arcs, cutting each other in C and D; draw C D, and it will cut A B in the middle point E.

[Prob]lem II.

At a given distance … *a straight line* C D, *parallel to a given straight line* …

… two points *m* and *r*, in … , with a distance equal to … the arcs *n* and *s*:—draw … these arcs, without cut- … nd it will be the parallel …

Note. This problem, as wel… …g one, is usually performed by an instrument called *the parallel rule*.

Problem III.

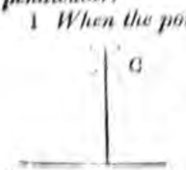
Through a given point r, to draw a straight line CD *parallel to a given straight line* A B.

From any point *n* in the line A B, with the distance *n r*, describe the arc *r m*:—from centre *r*, with the same radius, describe the arc *n s*:—take the arc *m r* in the compasses, and apply it from *n* to *s*:—through *r* and *s* draw C D, which is the parallel required.

Problem IV.

From a given point P *in a straight line* A B *to erect a perpendicular.*

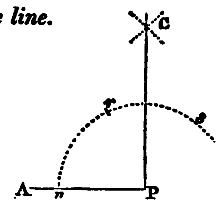
1 *When the point is in or near the middle of the line.*



On each side of the point P take any two equal distances, P *m*, P *n*; from the points *m* and *n*, as centres, with any radius greater than P *m*, describe two arcs cutting each other in C; through C, draw C P, and it will be the perpendicular required.

2. *When the point* P *is at the end of the line.*

With the centre P, and any radius, describe the arc $n\,r\,s$;—from the point n, with the same radius, turn the compasses twice on the arc, as at r and s:—again, with centres r and s, describe arcs intersecting in C:—draw C P, and it will be the perpendicular required.



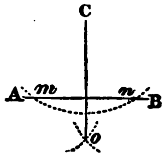
Note. This problem and the following one are usually done with an instrument called *the square.*

Problem V.

From a given point C *to let fall a perpendicular to a given line.*

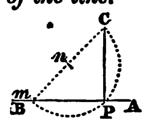
1 *When the point is nearly opposite the middle of the line.*

From C, as a centre, describe an arc to cut A B in m and n;—with centres m and n, and the same or any other radius, describe arcs intersecting in o: through C and o draw C o, the perpendicular required.



2. *When the point is nearly opposite the end of the line.*

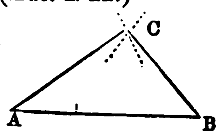
From C draw any line C m to meet B A, in any point m;—bisect C m in n, and with the centre n, and radius Cn, or mn, describe an arc cutting B A in P. Draw C P for the perpendicular required.



Problem VI.

To construct a triangle with three given right lines, any two of which must be greater than the third. (Euc. I. 22.)

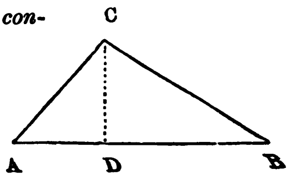
Let the three given lines be 5, 4 and 3 yards. From any scale of equal parts lay off the base A B = 5 yards; with the centre A and radius A C = 4 yards, describe an arc; with centre B and radius C B = 3 yards; describe another arc cutting the former arc in C:—draw A C and C B; then A B C is the triangle required.



Problem VII.

Given the base and perpendicular, with the place of the latter on the base, to construct the triangle.

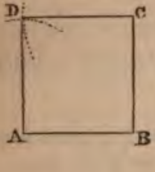
Let the base A B = 7, the perpendicular C D = 3, and the distance A D = 2 chains. Make A B = 7 and A D = 2;—at D erect the



perpendicular D C, which make = 3:—draw A C and C B; then A B C is the triangle required.

Problem VIII.

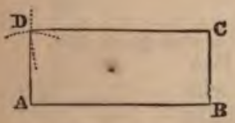
To describe a square, whose side shall be of a given length.



Let the given line A B be three feet. At the end B of the given line erect the perpendicular B C, (by Prob. IV. 2.) which make = A B:—with A and C as centres, and radius A B, describe arcs cutting each other in D: draw A D, D C, and the square will be completed.

Problem IX.

To describe a rectangled parallelogram having a given length and breadth.



Let the length A B = 5 feet, and the breadth B C = 2. At B erect the perpendicular B C, and make it = 2:—with the centre A and radius B C describe an arc; and with centre C and radius A B describe another arc, cutting the former in D: join A D, D C to complete the rectangle.

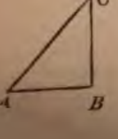
Problem X.

The base and two perpendiculars being given to construct a trapezoid.

Let the base A B = 6, and the perpendiculars A D and B C, 2 and 3 feet respectively. Draw the perpendiculars A D, D C, as given above, and join D C, thus completing the trapezoid.

Problem XI.

To construct a right angled triangle having a given base and perpendicular, and to find the hypothenuse.



Let the base A B = 6 feet, and the perpendicular B C = 8. Draw B C perpendicular to A B, and join A C; then A B C will be the triangle required, and A C being measured will be found = 10 feet.

Problem XII.

Having given the base and hypothenuse to construct the right angled triangle, and find the perpendicular.

(*See figure to last Problem.*)

Let A B = 6 feet and A C = 10.—Draw the perpendicular B C indefinitely; take A C = 10 feet in the compasses, and with one foot on A apply the other to C; join A C, which completes the triangle, and B C will be found = 8 feet.

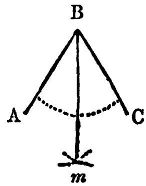
EXAMPLE.

A ladder 50 feet in length is placed with its foot 14 feet from a wall, the top of the ladder just reaching to the top of the wall; required the height of the wall.

Here 14 feet is the base of the right angled triangle, and 50 feet, = length of the ladder is the hypothenuse, with which the triangle being constructed, as in the last Problem, the perpendicular will be found = 48 feet.

Problem XIII.

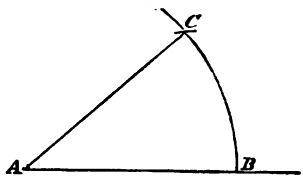
To divide a given angle A B C *into two equal parts.*



From the centre B, with any distance, describe the arc A C. From A and C, with one and the same radius, describe arcs intersecting in *m*. Draw the line B *m*, and it will bisect the angle as required.

Problem XIV.

To set off an angle to contain a given number of degrees.



Let the angle be required to contain 41 degrees. Open the compasses to the extent of 60° upon the line of chords, and setting one foot upon A, with this extent, describe an arc cutting A B in B; then taking the extent of 41° from the same line of chords, set it off from B to C; join A C; then B A C is the angle required.

Problem XV.

To measure an angle contained by two straight lines.

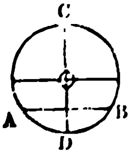
(*See last figure.*)

Let A B, A C contain the angle to be measured. Open the compasses to the extent of 60°, as before on the line of chords, and with this radius describe the arc B C, cutting A B, A C

produced, if necessary, in B and C; then extend the compasses from B to C, and this extent, applied to the line of chords, will reach to 41°, the required measure of the angle B A C.

A right angle, or perpendicular, may be laid off by extending the arc B C, and setting off the extent of 90° thereon. Also an angle greater than 90° may be laid off, by still further extending the arc, and laying the excess of the arc above 90°, from the end of the 90th degree.

NOTE. Angles are more correctly and expeditiously laid off and measured by an instrument called the protractor, to be hereafter described.



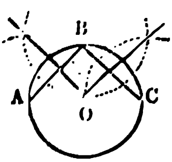
PROBLEM XVI.

To find the centre of a circle..

Draw any chord A B, and by Prob. I. bisect it perpendicularly with C D, which will be a diameter. Bisect C D in the point O, and that will be the centre.

PROBLEM XVII.

To describe the circumference of a circle through three given points A B C.



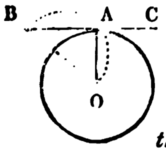
From the middle point B draw chords to the two other points A, C, bisect these chords perpendicularly by lines meeting in O, which will be the centre; then from the centre O, at the distance O A, or O B, or O C, describe the circle.

NOTE. In the same manner may the centre of an arc of a circle be found.

PROBLEM XVIII.

Through a given point A *to draw a tangent to a given circle.*

CASE I. *When* A *is in the circumference of the circle.*



From the given point A, draw A O to the centre of the circle; then through A draw B C perpendicular to A O, and it will be the tangent as required.

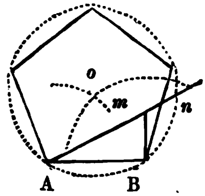
CASE II. *When the given point is* B *not in the circumference.*

From B draw B O to the centre of the circle; and on B O describe the semicircle B A O, cutting the circle in A: then B and A draw B A C, and it will be the tangent re-

PROBLEM XIX.

To make a regular pentagon on a given line A B.

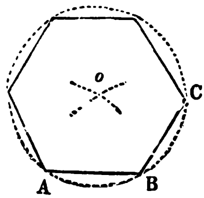
Make B *m* perpendicular and equal to half A B; draw A *m*, and produce it till *m n* be equal to B *m*; with centres A and B, and distance B *n* describe arcs intersecting in *o*, which will be the centre of the circumscribing circle: then with the centre *o*, and the same radius, describe the circle; and about the circumference of it apply A B the proper number of times.



PROBLEM XX.

To make a hexagon on a given line A B.

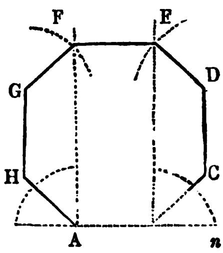
With the distance A B, and the centres A and B, describe arcs intersecting in *o*; with the same radius and centre *o* describe a circle, which will circumscribe the hexagon; then apply the line A B six times round the circumference, marking out the angular points, and connect them with right lines.



PROBLEM XXI.

To make an octagon on a given line A B.

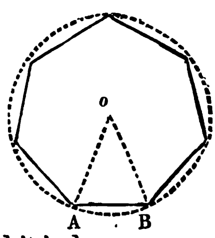
Erect A F and B E perpendicular to A B; produce A B both ways, and bisect the angles *m* A F and *n* B E with the lines A H and B C, each equal to A B; draw C D and H G parallel to A F or B E, and each equal to A B; with the distance A B, and centres G and D, cross A F and B E in F and E: then join G F, F E, E D, and it is done.



PROBLEM XXII.

To make any regular polygon on a given line A B.

Draw A *o* and B *o* making the angles A and B each equal to half the angle of the polygon, by Prob. XIV., with the centre *o* and distance *o* A describe a circle: then apply the line A B continually round the circumference the proper number of times, and it is done.



NOTE. The angle of any polygon, of which the angles o A B and o B A are each one half, is found thus: divide the whole 360 degrees by the number of sides, and the quotient will be the angle at the centre o; then subtract that from 180 degrees, and the remainder will be the angle of the polygon, and is double of o A B or of o B A. And thus you will find the numbers of the following table, containing the degrees in the angle o, at the centre, and the angle of the polygon, for all the regular figures from 3 to 12 sides.

No of sides.	Name of the Polygon.	Angle o at the centre.	Angle of the polygon.	Angle o A B or o B A.
3	Trigon . . .	120°	60°	30°
4	Tetragon . .	90	90	45
5	Pentagon . .	72	108	54
6	Hexagon . .	60	120	60
7	Heptagon . .	$51\frac{3}{7}$	$128\frac{4}{7}$	$64\frac{2}{7}$
8	Octagon . .	45	135	67
9	Nonagon . .	40	140	70
10	Decagon . .	36	144	72
11	Undecagon . .	$32\frac{8}{11}$	$147\frac{3}{11}$	$73\frac{7}{11}$
12	Dodecagon . .	30	150	75

PROBLEM XXIII.

In a given circle to inscribe any regular polygon; or to divide the circumference into any number of equal parts.

(*See the last figure.*)

At the centre o make an angle equal to the angle at the centre of the polygon, as contained in the third column of the above table of polygons: then the distance A B will be one side of the polygon, which being carried round the circumference the proper number of times, will complete the figure. Or, the arc A B will be one of the equal parts of the circumference.

PROBLEM XXII.

About a given circle to circumscribe any regular polygon.

Find the points m, n, p, &c., as in the last problem; to which draw radii m o, n o, &c., to the centre of the circle; then through these points m, n, &c., and perpendicular to these radii, draw the sides of the polygon.

EXAMPLE.

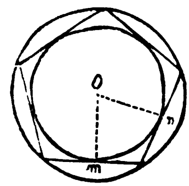
Let the radius of the given circle be five feet; then, having described a regular pentagon round it, the side of the figure

will be found = 7 feet $3\frac{1}{4}$ inches. If the figure to be described round the same circle be a regular hexagon, its side will be found = 5 feet $9\frac{1}{4}$ inches: and so on for any other regular polygons.

PROBLEM XXV.

To find the centre of a given polygon, or the centre of its inscribed or circumscribed circle.

Bisect any two sides with the perpendiculars *m o*, *n o*, and their intersection will be the centre; then with the centre *o*, and the distance *o m*, describe the inscribed circle; or with the distance to one of the angles as A, describe the circumscribing circle.

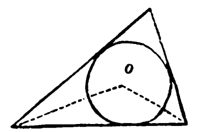


A

PROBLEM XXVI.

In any given triangle to inscribe a circle.

Bisect any of two of the angles with the lines A *o*, B *o*; and *o* will be the centre of the circle; then with the centre *o*, and radius the nearest distance to any one of the sides, describe the circle.



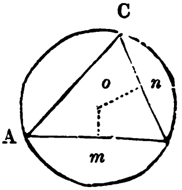
EXAMPLE.

Let the sides of the given triangle be 5, 4, and 3 feet; then, having inscribed a circle therein, its radius will be found = 1 foot.

PROBLEM XXVII.

About a given triangle to circumscribe a circle.

Bisect any of the two sides A B, B C, with the perpendiculars *m o*, *n o*,; with the centre *o*, and distance to any one of the angles, describe the circle.



EXAMPLE.

Let the sides of the given triangle be 15, 14, and 13 feet; then having described a circle about it, the radius will be found = 8 feet $1\frac{1}{8}$ inches.

PROBLEM XXVIII.

In, or about, a given square to describe a circle.

Draw the two diagonals of the square, and their inter-

section o will be the centre of both the circles: then with that centre, and the nearest distance to one side, describe the inner circle, and with the distance to one angle, describe the outer circle.

EXAMPLE.

Let the side of the given square be 3 feet: then, having described circles in and about it, the radius of the former will be found $= 1\frac{1}{2}$ feet, and that of the latter $= 2$ feet $1\frac{1}{2}$ inches nearly.

Problem XXIX.

In, or about, a given circle, to describe a square or an octagon.

Draw two diameters A B, C D, perpendicular to each other; then connect their extremities, and that will give the inscribed square A C D B. Also through their extremities draw tangents parallel to them, and they will form the outer square $m\ n\ o\ p$.

Note. If any quadrant, as A C, be bisected in q, it will give one-eighth of the circumference, or the side of the octagon.

Problem XXX.

In a given circle to inscribe a trigon, a hexagon, or a dodecagon.

The radius of the circle is the side of the hexagon; therefore from any point A in the circumference, with the distance of the radius, describe the arc B O F: then is A B the side of the hexagon; and therefore carrying it six times round will form the hexagon, or will divide the circumference into six equal parts, each containing 60 degrees.—The second of these, C, will give A C the side of the trigon, or equilateral triangle A C E, and the arc A C one-third of the circumference, or 120 degrees.—Also the half of A B, or A n, is one-twelfth of the circumference, or 30 degrees, which gives the side of the dodecagon.

Note. If tangents to the circle be drawn through all the angular points of any inscribed figure, they will form the sides of a like circumscribing figure.

EXAMPLE.

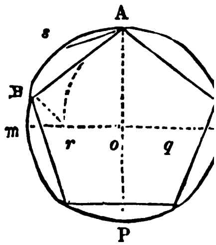
In a circle, the radius of which is 10 feet, inscribe a trigon, a hexagon, and a dodecagon.—Having measured a side of the

several figures, that of the trigon will be found = 17 feet inches, that of the hexagon 10 feet, and that of the dodecag 5 feet 2 inches.

Problem XXXI.

In a given circle to inscribe a pentagon or a decagon.

Draw the two diameters A P, *m n* perpendicular to each other, and bisect the radius *o n* at *q*; with the centre *q* and the distance *q* A, describe the arc A *r*; and with the centre A, and radius A *r*, describe the arc *r* B: then is A B one-fifth of the circumference; and A B carried five times over will form the pentagon. Also the arc A B bisected in *s*, will give A *s* the tenth part of the circumference, or the side of the decagon.



Note. Tangents being drawn through the angular points will form circumscribing pentagon or decagon.

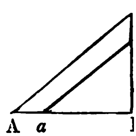
EXAMPLE.

In a circle, the radius of which is 10 feet, inscribe a pentag and decagon.—Having measured a side of each of the figur that of the pentagon will be found = 11 feet 9 inches, and th of the decagon = 6 feet 2 inches.

Problem XXXII.

To make a triangle similar to a given triangle A B C.

Make *a* B equal to the base of the required triangle; through *a* draw *a c* parallel to A C: then *a* B *c* is the triangle required.



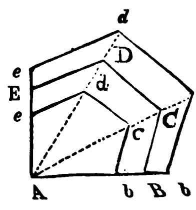
EXAMPLE.

Let A B = 12, A C = 15 and B C = 9 feet; and the side *a* B of the required triangle = 8 feet.—Then having drawn *a c* parallel to A C, the side *ac* will be found = 10, and B *c* = 6 fe

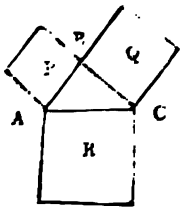
Problem XXXIII.

To make a figure similar to any other given figure A B C D E.

From any angle A draw diagonals to the other angles; take A *b* a side of the figure required; then draw *b c* parallel to B C, and *c d* to C D, and *d e* to D E, &c.



Problem XXXIV.



To make a square equal to two given squares P and Q.

Set two sides A B, B C, of the given squares, perpendicular to each other; join their extremities A C; so shall the square R, constructed on A C, be equal to the two P and Q taken together. (Euc. I., 47.)

Problem XXXV.

To make a square equal to the difference between two given squares, P, R.

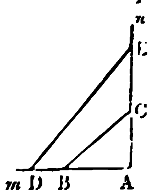
(*See the last figure.*)

On the side A C of the greater square, as a diameter, describe a semicircle; in which apply A B the side of the less square; join B C, and it will be the side of a square equal to the difference between the two P and R as required.

Problem XXXVI.

To make a square equal to the sum of any number of squares taken together.

Draw two indefinite lines A *m*, A *n*, perpendicular to each other at the point A. On the one of these set off A B the side of one of the given squares, and on the other A C the side of another of them; join B C, and it will be the side of a square equal to the two together. Then take A D equal to B C, and A E equal to the side of the third given square. So shall D E be the side of a square equal to the sum of the three given squares.—And so on continually, always setting more sides of the given squares on the line A *n*, and the sides of the successive sums on the other line A *m*.

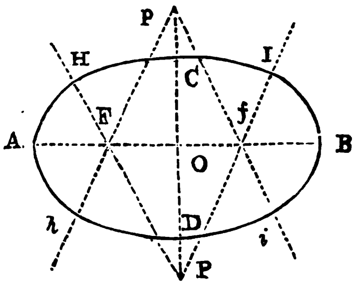


Problem XXXVII.

To construct a figure resembling an ellipse, by circular arcs from four centres.

On a line F *f*, of convenient length, describe two equilateral triangles F P *f*, F *p f*; prolong the sides of the triangles; join P *p*, as shown in the figure. With centres P, *p* and radius P H = *p h* describe the arcs H I, *h i*, meeting the prolonged sides of the triangles, and such that the diameter C D

equal to the required
›f the figure ; with
F, *f* and radius H F
= &c., describe the
A *h*, I *i*, and the
vill be completed.

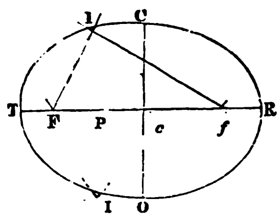


If the longer diameter
not obtained of the re-
ength by the above opera-
triangles F P *f*, F *p f*
enlarged or diminished,
isosceles, till by trials the
imensions are obtained.—This method of drawing the ellipse is prac-
the picture-frame makers.

Problem XXXVIII.

escribe a true ellipse.

T R be the trans-
C O the conjugate,
the centre. With
lius T *c* and centre
cribe an arc cutting
n the points F *f*;
are called the two
the ellipse.



me any point P in
nsverse ; then with
lii P T, P R, and centres F, *f*, describe two arcs inter-
in I ; which will be a point in the curve of the ellipse.
thus, by assuming a number of points P in the trans-
here will be found as many points in the curve as you
Then, with a steady hand, draw the curve through all
oints.

OTHERWISE,—WITH A THREAD.

e a thread of the length of the transverse, T R, and fasten
s with two pins in the foci F, *f*. Then stretch the thread,
will reach to I in the curve; and by moving a pencil
within the thread, keeping it always stretched, it will
ut the ellipse.

Problem XXXIX.

lescribe or construct a parabola.

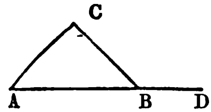
' being an absciss, and P Q its given ordinate ; bisect
n A, *join* A V, and draw A B perpendicular to it

makes the alternate angle equal, &c.: thus the angles A G H, G H D are equal; also the exterior angle E G B is equal to the interior and opposite G H D. (Euc. I. 29.)

THEOREM III.

The greatest side of every triangle is opposite the greatest angle. (Euc. I. 18.)

THEOREM IV.



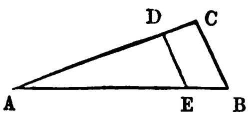
Let the side of A B *the triangle* A B C *be produced to* D, *the exterior angle* C B D *is equal to the interior angles at* A *and* C; *also the three interior angles of the triangle are equal to two right angles.* (Euc. I. 32.) Whence any two angles of a triangle being given the third becomes known.

THEOREM V.

(*See figure to Definition* 11.)

Let A B C be a right angled triangle, having a right angle at B; then, *the square on the side* A C *is equal to the sum of the square on the sides* A B, B C. (Euc. I. 47.) Whence any two sides of a right angled triangle being given the third becomes known.

THEOREM VI.



In any triangle A B C, let D E be drawn parallel to one of its sides, C B; then, A B *is to* A E *as* B C *is to* D E; and the triangles are said to be similar. (Euc. VI. 2.)

THEOREM VII.

(*See last figure.*)

Let A B C, A E D be similar triangles; then, *the triangle* A B C *is to the triangle* A E D *as the square* A B *is to the square of* A E: that is, similar triangles are to one another in the duplicate ratio of their homologous sides. (Euc. VI. 19.)

THEOREM VIII.

All similar figures are to one another as the squares of their homologous, or like, sides. (Euc. III. 20.)

THEOREM IX.

All similar solids are to one another as the cubes of their like *linear dimensions.* (Euc. VI. 24.)

EXPLANATION OF THE PRINCIPAL MATHEMATICAL CHARACTERS USED IN THIS WORK.

The sign for equality $=$ is read "equal;" thus 12 inches $=$ 1 foot.

The sign for addition $+$ is read "*plus* or more;" thus $2 + 3 = 5$. $a + b$, &c.

The sign for subtraction $-$ is read "*minus* or less;" thus $5 - 2 = 3$, $a - b$, &c.

The sign for multiplication $\times$ is read "into;" thus $5 \times 3 = 15$, $a \times b$, or $a\,b$, &c.

The sign for division $\div$ is read "by;" thus $15 \div 3 = 5$, or $\frac{15}{3} = 5$, or $\frac{a}{b}$, &c.

The signs for proportion, as : :: : "as, is to, so is, to;" thus as $2 : 5 :: 8 : 20$, or as $a : b :: c : d$, the fourth number being found by multiplying the second by the third, and dividing the first, or $\frac{5 \times 8}{2} = 20$, and $\frac{b\,c}{a} = d$.

The signs ( ) or $\{\ \}$ $\overline{}$ is called *vinculam* or brace: thus $(5 + 4) \times 2 = 9 \times 2 = 18$, or $\overline{5 + 4}\,2 = 18$, $(a + b, \times c$, or $\overline{a + b}\,c$, &c.

The signs ^{2}, ^{3}, &c., placed above a quantity, represent respectively the square, cube, &c., of that quantity; thus $5^2 = 5 \times 5 = 25$, $5^3 = 125$, $\overline{3 + 4}\,^2 = 7^2 = 49$, $4\,(5 + 3)^2 = 4 \times 8^2 = 256$; and a^2 and a^3 represent the square and cube of a, also $(a + b)^2\,c^3$ signifies that the square of the sum of a and b is to be multiplied by the cube of c, &c.

The sign $\surd$ or $\sqrt{}$ placed before a quantity, or $\frac{1}{2}$ placed above represents the square root of that quantity; thus $\sqrt{36} = 6$, $\sqrt{9 \times 16} = 12$, and $\sqrt{a \times b}$ or $\sqrt{a\,b}$ signifies the square root of the product of a and b, &c.

The sign $\sqrt[3]{}$ placed before a quantity, or $\frac{1}{3}$ placed above it, denotes the cube root of that quantity; thus

$\sqrt[3]{12 \times 2 \times 3 - 8}$, or $\sqrt[3]{(12 \times 2 \times 3 - 8)} = \sqrt[3]{72 - 8} = \sqrt[3]{64} = 4$, $\sqrt[3]{e\,\{(a + b)^2 - e\,d\}}$ denotes the cube root of the

difference of the square of the sum of a and b and the product of c and d multiplied into e. Also, the value of

$\sqrt[3]{e\{(a+b)^2 - c\,d\}}$, when $a = 2$, $b = 7$, $c = 5$, $d = 9$, and $e = 6$ is $\sqrt[3]{6\{(2+7)^2 - 5 \times 9\}} = \sqrt[3]{6\,(81 - 45)} = \sqrt[3]{6 \times 36} = \sqrt[3]{216} = 6$.

PART II.

MENSURATION OF LINES.

The Mensuration of Lines is applied to find the lengths of straight or curved lines, from the given lengths of other lines, on which these straight or curved lines depend.

TABLE OF LINEAL MEASURE.

Inches.	Feet.	Yards.	Poles.	Furlongs.	Mile.
12	1				
36	3	1			
198	$16\frac{1}{2}$	$5\frac{1}{2}$	1		
7920	660	220	40	1	
63360	5280	1760	320	8	1

$7\frac{23}{25}$ = 7·92 inches = 1 link.
22 yards = 4 poles = 1 chain of 100 links.
$69\frac{1}{2}$ English miles = 60 geographical miles = 1 degree.

Problem I.

To find one side of a right angled triangle, having the other two sides given.

The square of the hypothenuse is equal to both the squares of the two legs. (Euc. I. 47.) Therefore,

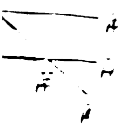
Rule I.—To find the hypothenuse; add the squares of the two legs together, and extract the square root of the sum.

Rule II.—To find one leg; subtract the square of the other leg from the square of the hypothenuse, and extract the square root of the difference.

STEREOGRAPHICAL THEOREMS.

(Necessary to be known by Beginners.)

THEOREM I.



Angles vertically opposite are equal; thus the angle E G L = [illegible] H [illegible] and E G B = A G K [illegible]

THEOREM II.

(See last figure.)

line E F cutting two parallel right lines A B, C D [illegible]

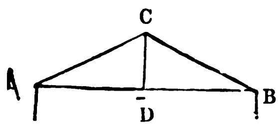
3. A ladder is to be placed so as to reach the top of a wall $33\frac{3}{4}$ feet high, and the foot of the ladder cannot be placed nearer the wall than 18 feet; what must be the length of the ladder?
Ans. $38\frac{1}{4}$ *feet.*

4. The side of a square is 100 yards; what is the length of its diagonal? *Ans.* 141·4 *yards.*

5. A line of 320 feet will reach from the top of a precipice, standing close by the side of a river, to the opposite bank: required the breadth of the river; the height of the precipice being 103 feet. *Ans.* 302·97 *feet.*

6. A ladder of 50 feet long being placed in a street, reached a window 28 feet from the ground on one side; and by turning the ladder over, without removing the foot out of its place, it touched a moulding 36 feet high on the other side: required the breadth of the street. *Ans.* 76·123 *feet.*

7. The width of a house is 48 feet, and the height of ridge above the side walls 10 feet; required the length of one of the rafters.



In the annexed figure A B is the width of the house, or length of the tie-beam of the rafters A C, B C; and C D the height of the ridge or length of the king-post; and since D is the middle point of A B, we shall have $AD = \frac{1}{2} AB = 24$ feet. Whence by the first formula;

$$AC = \sqrt{AD^2 + CD^2} = \sqrt{24^2 + 10^2} = \sqrt{676} = 26 \text{ feet},$$

the required length of one of the rafters.

8. Required the height of an equilateral triangle, the side of which is 10 feet. *Ans.* 8 *feet* 8 *in. nearly.*

9. The base of an isosceles triangle is 25 feet, and its two sides are each $32\frac{1}{2}$ feet; required the perpendicular. *Ans.* 30 *feet.*

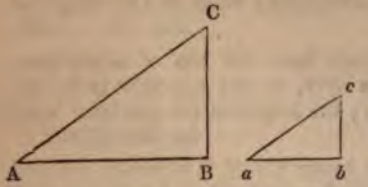
10. The diagonal of a square is 10 yards, required the length of one of its sides. *Ans.* 7 *yds.* 0 *feet* $2\frac{1}{2}$ *in.*

11. A ladder, standing upright against a wall 100 feet high, was pulled out at the foot 10 feet from the wall; how far did the top of the ladder fall? *Ans* 6 *inches nearly.*

12. The upright axle of the horse-wheel of a thrashing machine is placed with its centre $3\frac{1}{2}$ yards from a wall; but the shafts of the axle are 5 yards in length, measured from the centre: how much of the wall must be removed to admit it to revolve?
Ans. 7 yds. 0 ft. 5 in.

Problem II.

Having given any two of the dimensions of the figure A B C, and one of the corresponding dimensions of a similar figure a b c, to find the other corresponding dimension of the last figure.



Rule.—Let A B C, $a\ b\ c$ be two similar triangles, then by Theorem VI., page 17.

$$AB : BC :: ab : ac, \text{ or } ab : ac :: AB : BC.$$

The same proportion holds with respect to the similar lineal parts of any other similar figures, whether plane or solid.

EXAMPLES.

1. The shadow of a cane 4 feet long, set perpendicularly, is 5 feet, at the same time that the shadow of a lofty tree was found to be 83 feet; required the height of the tree, both shadows being on level ground.

Let $b\,c$ be the cane, and B C the tree, their shadows being respectively represented by $a\,b$ and A B: the upper extremities of the cane and tree being joined with the extremities of their shadows, giving the parallel lines $a\,c$, A C for the directions of the sun's rays, and thus constituting similar triangles $a\,b\,c$, ABC: whence $ab : bc :: AB : BC$,

that is $5 : 4 :: 83 : 66\frac{2}{5}$

$$\begin{array}{r} 83 \\ 4 \\ \hline 5)332 \\ \hline 66\frac{2}{5} \end{array}$$

$66\frac{2}{5}$ feet $=$ BC, the height of the tree.

2. The side of a square is 5 feet, and its diagonal 7·071 feet, what will be the side of a square, the diagonal of which is 4 feet.?
Ans. 2 *ft.* 10 *in. nearly.*

3. In the ground plan of a building 120 feet long and 50 broad, the length, as laid down, is 10 inches; what must be its breadth?
Ans. $4\frac{1}{6}$ *inches.*

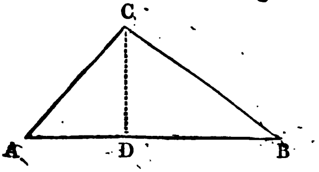
4. The scale of the Ordnance survey of Ireland is 6 inches to 1 mile, what length of paper will be sufficient for the map of that country, its length being 300 miles?
Ans. 50 *yards.*

5. The length of the shadow of the Monument (London) is 51¼ feet, while the shadow of a post 4 feet high, is 3 feet; required the height of the Monument. *Ans.* 202 *feet.*

Problem III.

The two sides and the base of a triangle (A B C) *are given to find the perpendicular* (C D).

Rule.—The segments of the base A D, D C must be first found. Let B C be the greater of the two sides, then B. D will be the greater of the two segments. Then, as the base A B is to the sum of the sides B C + C A, so is the difference of the sides B C—C A to the difference of the segments of the base B D — D A. Half this difference, being added to and subtracted from half the base A B, will give respectively the segments B D, D A; though only one of the segments is required to be found. Now, either of the sides and its adjacent segments constitute a right angled triangle, whence the perpendicular C D may be found by Rule II., Prob. I.



Formulæ.

Put A B $= a$, B C $= b$ and C′ A $= c$; then from the proportion in the Rule

$$BD - DA = \frac{b^2 - c^2}{a}; \text{ whence}$$

$$BD = \tfrac{1}{2}\left(a + \frac{b^2 - c^2}{a}\right), \text{ and}$$

$$DA = \tfrac{1}{2}\left(a - \frac{b^2 - c^2}{a}\right).$$

EXAMPLES.

1. The three sides of a triangle are 42, 40, and 26 feet; required the perpendicular on the longest side.

By the Rule

AB : BC + CA : : BC — CA : BD — DA, that is,
42 : 66 : : 14 : 22, and
$\frac{1}{2}(42 - 22) = 10$ feet $=$ A D

Or by the last Formula

$$DA = \tfrac{1}{2}\left(42 - \frac{40^2 - 26^2}{42}\right) = 10 \text{ feet},$$

and $CD = \sqrt{AC^2 - DA^2} = \sqrt{26^2 - 10^2} = 24$ feet.

2. The base of a triangle is 30, and the two sides 25 and 35; required the perpendicular. *Ans.* 24 *feet* 6 *in. nearly.*

3. A house 21 feet in width, has a roof with unequal slopes, the lengths of which, from the eaves to the ridge, are 20 and 13 feet; required the height of the ridge above the eaves.

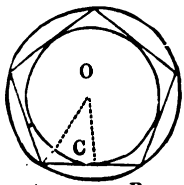
Ans. 12 *feet.*

NOTE. All the preceding examples may be readily solved by construction, by first laying down the triangles, as in Prob. VI., *Practical Geometry*, and then letting fall the perpendicular, as in Prob. V.

PROBLEM IV.

The side A B *of a regular polygon being given to find the radii* O C *and* O A *of its inscribed and circumscribed circles.*

RULE.—Multiply the side of the polygon by the number opposite its name in the following Table, in the column headed "Rad. Inscribed Circle," or in that headed "Rad. Circumsc. Circle," accordingly as the one or the other radius may be required.



FORMULÆ.

Let r and R be the radii of the inscribed and circumscribed circles respectively, q and p their respective tabular radii, and l = side of the polygon; then $r = l\,q$, and $R = l\,p$; also

$$l = \frac{r}{p} = \frac{q}{R}.$$

TABLE OF POLYGONS.

No. of Sides.	Name.	Rad. Inscribed Circle.	Rad. Circums. Circle.	Area.
3	Trigon or equi-triangle .	·2887	·5773	·4330
4	Tetragon or square . .	·5000	·7071	1·0000
5	Pentagon	·6882	·8506	1·7205
6	Hexagon	·8660	1·0000	2·5981
7	Heptagon	1·0383	1·1524	3·6339
8	Octagon	1·2071	1·3066	4·8284
9	Nonangon	1·3737	1·4619	6·1818
10	Decagon	1·5388	1·6180	7·6942
11	Undecagon	1·7028	1·7747	9·3656
12	Dodecagon	1·8660	1·9319	11·1962

By the Rule

$$
\begin{array}{rl}
 & 24 = \tfrac{1}{2}\,BE = BD \\
 & 24 \\ \hline
CD = 18) & 576 \\ \hline
 & 32 \\
 & 18 = CD \\ \hline
 & 50 \text{ feet} = AC.
\end{array}
$$

whence the radius B O = 25 feet.

2. The span (chord) of the iron arch of Sunderland bridge is 240 feet, and the rise (height) of the crowns of the arch 34 feet: with what radius was the arch drawn?

By Formula (1).

$$\text{The diameter} = \frac{120^2}{34} + 34 = 440{\cdot}41 \text{ feet.}$$

whence the required radius $= 440{\cdot}41 \div 2 = 220{\cdot}205 = 220$ feet $2\tfrac{1}{2}$ inches.

3. On a parliamentary map of 4 chains to an inch, the chord of a railway curve measured 40 inches, and its height 5 inches, re. quired the radius of the curve both on the map and on the ground-

$$\text{The diameter} = \frac{20^2}{5} + 5 = 85 \text{ inches, whence}$$

the radius $= 85 \div 2 = 42\tfrac{1}{2}$ inches on the map.

And, since the scale of the map is 4 chains to an inch, we shall have $42\tfrac{1}{2} \times 4 = 170$ chains $= 2\tfrac{1}{8}$ miles, the radius of the curve on the ground.

4, The chord of the whole arc is 48 feet, and its height 7; required the chord of half the arc.

By Formulæ (2).

$c = \sqrt{C^2 + v^2} = \sqrt{24^2 + 7^2} = 25$ feet, the required chord.

5. The chord of half the arc of a bridge is 24 feet, and the rise of the crown of the arc 16 feet; required the radius of the circle of which the arch is a part.

Ans. By formula (3) the diameter is found = 36 feet, whence the required radius is 18 feet.

6. A circular grass plot of 100 yards diameter is cut by a walk through the centre, this walk is cut at right angles by another walk through the middle of the radius; required the length of the last named walk.

By transposing formula (1) $C = \sqrt{v\,(d - v)} = \sqrt{25\,(100 - 25)}$ $= 43{\cdot}3$ yards the double of which is the length of the walk.— The same result may be obtained from the right angled triangle B D O.

7. The rise of the circular arch of a bridge is 12 feet and the radius of the whole circle is 100 feet; required the distance from the spring of the arch to the crown, viz., the chord of half the arch. *Ans.* 49 *feet nearly.*

PROBLEM VII.

To find the length of any arc of a circle.

CASE I.—*When the degrees in the arc and the radius are given.*

RULE I.—As 180° is to the number of degrees in the arc, so is 3·1416 times the radius to its length.

CASE II.—*When the chord of half and the whole arc are given.*

RULE II.—From 8 times the chord of half the arc subtract the chord of the whole arc, and take $\frac{1}{3}$ of the remainder for the length of the arc nearly.

FORMULÆ. (*See last figure.*)

Put r = radius B O, $\Delta = 180°$, δ = degrees in the arc B E, and $\pi = 3·1416$, and l = length of the arc; then

$$l = \frac{r\,\delta\,\pi}{\Delta}, \text{ and } r = \frac{l\,\Delta}{\delta\,\pi}$$

EXAMPLES.

1. To find the length of an arc of 30 degrees, the radius being 9 feet.

By Rule I.

$$\begin{array}{r} 3·1416 \\ 9 \\ \hline \end{array}$$

$$180 : 30 :: 28·2744 : 4·7124 \text{ feet.}$$

By first Formula $l = \dfrac{9 \times 30 \times 3·1416}{180} = \dfrac{3 \times 3·1416}{2} = 4·7124$

2. The length of the arc of a circle of 30 degrees is 9 feet 5 inches, required its radius.

Ans. By the second formula, 18 *feet nearly.*

3. The chord B E of the whole arc being 4·65374 feet, and the chord B C of the half arc 2·34947; required the length of the arc.

By Rule II.

$$\begin{array}{r} 2·34947 \\ 8 \\ \hline 18·79576 \\ 4·65874 \\ \hline 3)14·13702 \\ \hline \end{array}$$

Ans. 4·71234 *feet.*

3. Required the length of an arc of 12 degrees 10 minutes, or $12\frac{1}{6}$ degrees, the radius being 10 feet.

By Rule I., 2·1234 *feet, Ans.*

4. Required the length of the iron arch, in example 2, Prob. VI.

First, the chord of $\frac{1}{2}$ the arch, or distance from spring to crown, by Formula 2, Prob. VI., will be found 124·724 feet. Whence, by Rule II. of this Problem, we shall have the required length of the arch = 252 feet 7 inches.

5. Find the length of one of the arcs of the six equal segments of an iron girder, the whole span of the arch being 120 feet, and the radius 180. *Ans.* 20 *feet* 4·67 *inches.*

Rule III. is not sufficiently accurate for finding the length of the arc, when it is greater than $\frac{1}{4}$ of the circumference of the circle: in such cases, (see figure to Prob. VI.) the chord of $\frac{1}{4}$ of the arc B C E = chord of $\frac{1}{2}$ the arc B C (not shown in the figure) must be found by the formula.

$$\text{Chord of } \tfrac{1}{4} \text{ of arc B C E} = \sqrt{\tfrac{1}{2}\, d\,(d - \sqrt{d^2 - c^2})}.$$

in which d and c are the same as in Prob. VI.; after which Rule II. may be applied with sufficient accuracy to find the length of the $\frac{1}{2}$ arc B C, which, being doubled, will give the whole length B C E.

6. Required the length of a circular iron girder, the span (B E) of which is 48 feet, and the rise (C D) at the crown 18 feet.

By Formulæ 1 and 2, Problem VI., d = A C is found = 50 feet, and c = B C = 30; whence, by the formula just given, the chord of $\frac{1}{4}$ of arc B C E = $\sqrt{25\ (50 - \sqrt{50^2 - 30^2})}$ = 15·8113, and by Rule II., (15·8113 × 8 — 30) ÷ 3 = 32·1635 feet = arc B C, the double of which is 64·3270 feet = the required length of the arch B C E. But by using Rule II., without the above formula, the length of the arch is found to be 64 feet, or nearly 4 inches short of its more accurate length, as previously found.

NOTE. The true method of finding the length of an arc of a circle is to find the natural sine of the angle B O D (figure to Prob. VI.) and its corresponding number of degrees, minutes, &c., which, being doubled, give the angular measure of the whole arc B C E; whence the length of the arc may be accurately found by Rule I. But the first part of this operation is the province of Trigonometry; *moreover, sufficient accuracy for all practical purposes may be obtained by Rule II. for arcs* less than a quadrant; and the accuracy [illegible]

secured by means of the formula used in Example 6, in cases where the arc approaches near to a semicircle.

Remark.—When the arc is greater than a semicircle, the remaining part of the circumference must be found by Rule II., with the help of the formula used in Example 6, if necessary. This remark does not apply to cases where the degrees of the arc are given, which are solved by Rule I.

Problem VIII.

To find the diameter of a circular zone, its two parallel chords A B, C D, *and its breadth* E F, *being given.*

This and the following Problems may be omitted by the student, as not being much required in practice.

FORMULÆ.

Let C and c be the half chords C F and A E respectively, b the breadth E F, and d the diameter K L = twice radius O B; then

$$d = \sqrt{\left\{b^2 + 2\,(C^2 + c^2) + \left(\frac{C^2 - c^2}{b}\right)^2\right\}}.$$

$$\text{Also } AC = BD = \sqrt{\left(b^2 + \overline{C - c}^2\right)}, \text{ and}$$

$$GH = \tfrac{1}{2}d - \tfrac{1}{2}\sqrt{\left\{(C + c)^2 + \left(\frac{C^2 - c^2}{b}\right)^2\right\}}.$$

EXAMPLES.

1. The parallel sides of a circular zone are 6 and 8 feet, and its breadth 7 feet; required the diameter of the circle.

By the first Formulæ the diameter

$$d = \sqrt{\left\{7^2 + 2\,(4^2 + 3^2) + \left(\frac{4 - 3^2}{7}\right)^2\right\}} = \sqrt{49 + 50 + 1} = 10 \text{ ft.}$$

2. Find the chord B D and the height G H of the zone in the preceding Example.

Here, the diameter d is first found, as above; then by the second and third Formulæ,

$$BD = \sqrt{(7^2 + 4 - 3^2)} = \sqrt{49 + 1} = 7{\cdot}07 \text{ feet, and}$$

$$GH = \tfrac{1}{2}\,10 - \tfrac{1}{2}\sqrt{\left\{(4 + 3)^2 + \left(\frac{4^2 - 3^2}{7}\right)^2\right\}} = 5 - \tfrac{1}{2}\sqrt{49 + 1} = 1{\cdot}465 \text{ ft.}$$

3. The parallel chords of a zone are the same as in the Ex-ample 1, and its breadth 1 foot; required the diameter.

Ans. 14 feet.

NOTE. In this example the two chords are both on the same side of the centre of the circle.

4. The two parallel chords of a circular zone are 16 and 12 feet, and the diameter of the circle 20 feet; required the breadth of the zone. *Ans.* 14 *feet.*

NOTE. 1. The breadth of the zone, in this example, is found by squaring and transposing the first formula, whence there results a quadractic equation, from which the value of b is found.

NOTE 2. When the chord B D = A C, and the height G H have been found, the lengths of the equal arcs A C, B D are found by the Prob. VII.

PROBLEM IX.

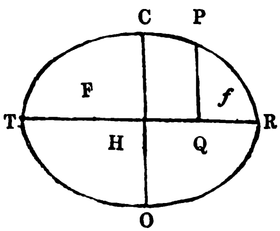
In an ellipse are given any three of the four following parts to find the fourth, viz. the transverse axis T R, *the conjugate axis* C O, *the abscissa* H Q, *and the ordinate* P Q.

FORMULÆ.

Put a = semitransverse = H R, b = semiconjugate = C H, x = abscissa = H Q, and y = ordinate = P Q; then

$$x = \frac{a}{b}\sqrt{b^2 - y^2},\ y = \frac{b}{a}\sqrt{a^2 - x^2},$$

$$a = \frac{b\,x}{\sqrt{b^2 - y^2}},\ \text{and } b = \frac{a\,y}{\sqrt{a^2 - x^2}}$$



Also the focial distance from the centre.

$$\text{H F} = \text{H}f = \sqrt{a^2 - b^2}.$$

EXAMPLES.

1. The transverse axis is 30, the conjugate 20, and the abscissa 3 feet.

By the second formula,

$$\text{P Q} = y = \tfrac{10}{15}\sqrt{15^2 - 3^2} = 9{\cdot}798 \text{ feet.}$$

2. The transverse T R = 70 feet, the conjugate C O = 50, and the ordinate P Q = 20; required the abscissa H Q.

Ans. By the first formula, H Q = 21 *feet.*

3. The transverse is 180 inches, the ordinate 16, and the abscissa 54; required the conjugate.

Ans. By the fourth formula, the conjugate = 40 *inches.*

4. If the conjugate be 50 feet, the ordinate 20, and the abscissa 21; what is the transverse?

Ans. By the third formula, the transverse = 70 feet.

5. The transverse T R $= 100$ yards, and the conjugate C O $= 60$; required the distance of the foci F f from the centre H.

Ans. By the last formula, H F $=$ H $f = 40$ *yards.*

6. The ratio of the major and minor axes of the earth's orbit is as 1 to n, the former being about 190,000,000 miles $= 2\,a$, How much is the earth nearer to the sun in winter than in summer?

Ans. The distance here required is twice the focial distance from the centre of the earth's elliptical orbit, which, by the last formula is found to be $2\,a\sqrt{1 - n^2}$.

7. Required the distance of the foci of an elliptical section, passing through the poles of the earth, the earth's axes being 7926 and 7899 miles.

Ans. 654 *miles, or* 327 *miles each from the earth's centre.*

Problem X.

The axes of an ellipse are given to find its circumference.

Rule I.—Multiply half the sum of the two axes by 3·1416, and the product will give an approximate length of the circumference, *which will be found near enough for most practical purposes.*

Rule II.—To half the sum of the two axes add the square root of half the sum of their squares, and multiply half the sum by 3·1416 for the circumference *very nearly.*

FORMULÆ (*see last figure*).

Let $2\,a$ and $2\,b$ represent the axes, as in the last problem, and $\pi = 3·1416$; then,

Circumf. $= \pi\left(a + b\right)$, or $= \frac{1}{2}\,\pi\left(a + b + \sqrt{2\,(a^2 + b^2)}\right)$.

EXAMPLES.

1. The axes of an ellipse are 15 and 10 feet; required the circumference by Rule I.

Ans. 39 *feet* 3 *inches.*

2. The axes being the same as in the last example; required the circumference by Rule II.

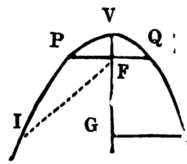
Ans. 39 *feet* 8 *inches nearly.*

3. Find the meridional circumference of the earth, the axes as given in the last example of Prob. IX.

Ans. 24,858 *miles nearly.*

PROBLEM XI.

In a parabola I V H, *the focus of which is* F, *any two of the three following parts, viz., the parameter* P Q, *the abscissa* V G, *and the ordinate* G H *being given, to find the third part.*



FORMULÆ.

Put P Q = parameter $= p$, V G = abscissa $= x$, and C H = ordinate $= y$; then

$$x = \frac{y^2}{p},\ y = \sqrt{p\,x},\ \text{and}\ p = \frac{y^2}{x}.$$

EXAMPLES.

1. The parameter P Q of a parabola is 50, and its ordinate G H = 60 feet, required the abscissa V G.

Ans. By the first formula x = G H = $\frac{60^2}{50} = 72$.

2. The parameter of a parabola is 10, and its ordinate 4; required the abscissa. *Ans.* 1·6.

3. The abscissa of a parabola is 4, and its corresponding ordinate 10; required the parameter. *Ans.* 25.

PROBLEM XII.

To find the length of the arc of a parabola, its ordinate and abscissa being given. (*See last figure.*)

FORMULÆ.

Let x and y represent the same parts, as in the last Problem; then The $\frac{1}{2}$ arc V H $= \sqrt{\frac{4}{3}\,x^2 + y^2}$ nearly

EXAMPLES.

1. Required the half arc V Q of a parabola, V F being = 3 feet, and F Q = 6.

Ans. V Q $= \sqrt{\frac{4}{3}\,3^2 + 6^2} = 6$ *feet* $11\frac{1}{7}$ *inches.*

2. The abscissa is 2, and the ordinate 6; required the length of the half arc of the parabola. *Ans.* 6.4291.

NOTE. 1. The parabola is the path of projectiles *in vacuo;* it is also used in the astronomical theory of comets.

NOTE 2. The student who wishes for further information concerning this curve, as well as concerning the ellipse and hyperbola, may consult the various works on *conic sections.*

2. How many square feet contains the plank, whose len 12 feet 6 inches, the breadth at the greater end 1 foot 8 i and at the less end 11 inches? *Ans* $13\frac{1}{4}\frac{1}{4}$ *J*

3. Required the area or a trapezoid, the parallel sides be feet 3 inches and 18 feet 6 inches, and the distance betweer 8 feet 5 inches. *Ans.* 167 *square feet,* 3′ 4″

PROBLEM IV.

To find the area of a trapezium.

CASE I.—*For any trapezium.*

Divide it into two triangles by a diagonal; then find th of these triangles, and add them together.

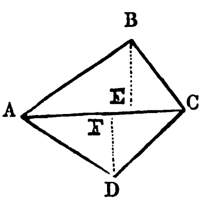
Or, if two perpendiculars be let fall on the diagonal, fro other two opposite angles, the sum of these perpendiculars multiplied by the diagonal, half the product will be the a the trapezium.

CASE II.—*When two opposite angles are supplements o other.*

Add all the four sides together, and take half the sum subtract each side separately from the half sum; then mu the four remainders continually together, and take the squa of the last product for the area of the trapezium.

EXAMPLES.

1. To find the area of the trapezium A B C D, the di A C being 42, the perpendicular B E 18, and the perpenc D F 16.



18
16
—
34 Sum
42
—
68
136
—
2)1423
714 *Ans.*

In the trapezium A B C D, the side A B is 15, D D 12, and the diagonal A C is 16: require

AC 16				AC 16			
AB 15				CD 14			
BC 13				AD 12			
2)44				2)42			
22	22	22	half sum	21	21	21	half sum
16	15	13		16	14	12	
6	7	9		5	7	9	
7				7			
42				35			
9				9			
378				315			
22				21			
756				315			
756				630			

$\sqrt{8316}=91{\cdot}1921$ $\sqrt{6615}=81.3326$

The triangle A B C .... 91·1921
The triangle A B C .... 81·3326

The trapezium A B C D 172·5247, *Ans.*

3. If a trapezium have its opposite angles supplements to each other, and have four sides 24, 26, 28, 30; required its area.

By Rule II. the area is 723·989.

4. How many square yards of paving are in the trapezium, the diagonal of which is 65 feet, and the two perpendiculars let fall on it 28 and 38·5 feet? *Ans.* $222\frac{1}{12}$ *yards.*

5. What is the area of a trapezium, the south side being 27·40 chains, east side 35·75 chains, north side 37·55 chains, west side 41·05 chains, and the diagonal from south-west to north-east 48·35 chains? *Ans.* 123*a*. 0*r*. 11.8656*p*.

6. What is the area of a trapezium, the diagonal of which is $108\frac{1}{2}$ feet, and the perpendiculars $65\frac{1}{4}$ and $60\frac{3}{4}$ feet.

Ans. $705\frac{1}{4}$ *square yards.*

7. What is the area of a trapezium, the four sides being 12, 13, 14, 15? having its opposite angles supplemental.

Ans. 180·997.

8. In the four sided field A B C D, on account of obstructions in the two sides A B C D, and in the perpendiculars B F, D E, the following measures only could be taken: namely, the two sides B C 265 and A D 220 yards, the diagonal A C 378 yards,

and the two distances of the perpendiculars from the ends of the diagonal, namely A E 100, and C F 70 yards: required the area in acres, when 4840 square yards make an acre.

Ans. 17*a.* 2*r.* 21*p.*

9. When A B = 314, B C = 232, C D = 228½, D A = 266½, and the diagonal A C = 417½ feet; required the area in square yards. *Ans.* 8641⅑ *square yards.*

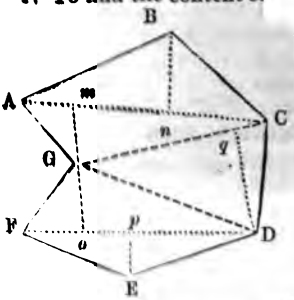
Problem V.

To find the area of an irregular polygon.

Rule.—Draw diagonals dividing the figure into trapeziums and triangles; then find the areas of all these separately, and add them together for the con[illegible] whole figure.

Ex

1. To find the content of [illegible] figure A B C D E F G A, [illegible] are given the following [illegible] gonals and perpendiculars [illegible] mely,



[illegible]	5·5
[illegible]	5·2
[illegible]	4·4
[illegible]	1·3
B*n*	1·8
G*o*	1.2
E*p*	0·8
D*q*	2·3

1st, For trapez. ABCG.	2nd, For trapez. GDEF.	3rd For triangle GCD.
1·3	1·2	4·4
1·8	0·8	2·3
3·1	2·0	132
5·5	5·2	88
1·55	10·4	10·12
15·5		
17·05		

17·05 double A B C G
10·40 double G D E F
10·12 double G C D

2)37·57 double the whole.

18·785

2. Required the area of the figure A B C D E F G, when $AC = 12$, $FD = 11$, $GC = 9\frac{1}{2}$, $Gm = 3\frac{1}{4}$, $Bn = 4$, $Go = 2\frac{1}{2}$ $Ep = 1\frac{1}{3}$, and $Dq = 4\frac{1}{2}$ feet.

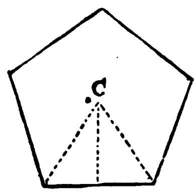
PROBLEM VI.

To find the area of a regular polygon.

RULE I.—Multiply the sum of the sides or perimeter of the polygon by half the perpendicular from its centre to one of its sides, and the product will be the area.

RULE II.—Multiply the square of the side of the polygon by the number opposite its name, in the column headed "Areas," in the Table to Prob. IV., Part II. and the product will be the area.

FORMULÆ.



Let $s = AB =$ side of the polygon, $p = CP$ perpendicular from the centre on A B, $n =$ number of sides of the polygon, and $a =$ its tabular area; then $A = \frac{1}{2} n p s$, and $A. = a s^2$. Also $s = \sqrt{\frac{A}{a}} = \frac{2A}{np}$, and $p = \frac{2A}{ns}$.

EXAMPLES.

1. Required the area of a regular pentagon, the side A B of which is 25 feet, and the perpendicular $CP = 17{\cdot}205$.

By Rule I.

$$\begin{array}{r} 17{\cdot}205 \\ 25 \times 5 = 125 = \text{perim.} \\ \hline 86025 \\ 34410 \\ 17205 \\ \hline 2)2150{\cdot}625 \\ \hline 1075{\cdot}3125 \textit{ sq. feet.} \end{array}$$

By Rule II.

$$\begin{array}{r} 1{\cdot}7205 \text{ table area.} \\ 625 = 25^2 \\ \hline 86025 \\ 34410 \\ 103230 \\ \hline 1075{\cdot}3125 \textit{ sq. feet. Ans.} \end{array}$$

2. To find the area of the hexagon, the side of which is 20 feet. *Ans.* 1039·23 *square feet.*

3. To find the area of the trigon, or equilateral triangle the side of which is 20 feet. *Ans.* 173·205 *square feet.*

4. Required the area of an octagon, the side of which is 20 feet. *Ans.* 1931·37 square feet.

5. W[illegible] is the area of a decagon, the side of which i[illegible] feet.

Ans. 3077·68 *square fe[et]*

6. Required the side of a decagon, the area of which is [illegible] square feet.

By the third formula, the side $s=\sqrt{\frac{A}{a}}$, that is,

$$\sqrt{\frac{16}{7\cdot6942}} = 1\cdot442 \textit{ feet} = 1 \textit{ foot } 5\cdot3 \textit{ inches. Ans.}$$

7. The fence of an octagonal inclosure, within a square in [a] city, cost £840 at 4*s.* 8*d.* per foot; what will be the cost of th[e] gravelling the surface at $10\frac{1}{4}$*d.* per square yard?

Ans. £132 0*s.* $6\frac{1}{2}$*d.*

8. The corners of a square are cut off so as to form an octagon; required the area of the octagon, the side of the square being 200 feet.

Ans. 3681·8 *square yards.*

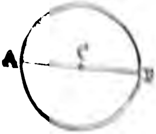
Problem VII.

To find the area of a circle when the radius, or half diameter is given.

Rule I.—Multiply the square of the radius by 3·1416 for the area.

To find the area of a circle when the circumference is given.

Rule II.—Multiply the square of the circumference by ·07958.



Put the radius A C $= r$, the circumference $= c$, and $3\cdot1416 = \pi$; then

$$A = \pi r^2, \text{ and } r = \sqrt{\frac{A}{\pi}}; \text{ also}$$

$$A = \frac{c^2}{4\pi} = \tfrac{1}{2} r c, \text{ and } c = \sqrt{4A\pi},$$

EXAMPLES.

1. Required the area of a circle, the radius of which is [illegible] feet.

By Rule I., or the first formula,

$3\cdot1416 \times 5^2 = 3\cdot1416 \times 25 = 78\cdot54$ square feet.

2. The circumference of a circle is 154 feet; what [illegible]

3. A circular piece [illegible]

exactly an acre, required the length of the chord with which the circle must be traced.

By the second formula, the length of the chord, or

$$r = \sqrt{\frac{4840}{3\cdot1416}} = 39\tfrac{1}{4} \text{ yards very nearly.}$$

4. How many square yards are in a circle whose diameter is $3\frac{1}{2}$ feet? *Ans.* 1·069.

5. How many square feet does a circle contain, the circumference being 10·9956 yards. *Ans.* 86·19543.

6. The area of the piston of a steam engine is required to be 1192 square inches to give it the requisite power; required the interior diameter of the cylinder, and its exterior circumference the thickness of the metal being one inch.

Ans. { *Interior diameter* 39 *inches nearly.* / *Exterior circumference* 10 *feet* $8\frac{3}{4}$ *inches.* }

7. The circumference of the circular paling of a plantation was found to be $235\frac{1}{8}$ yards, what is its area.

Ans. 4400 *square yards.*

8. What is the circumference of a circle, the area of which is an acre? *Ans.* 246 *yards* 1 *foot* $10\frac{1}{3}$ *inches.*

Problem VIII.

To find the area of a sector of a circle.

Rule I.—Multiply the radius, or half the diameter, by half the arc of the sector, for the area. Or, multiply the diameter by the arc of the sector, and take $\frac{1}{4}$ of the product.

Note. The arc may be found by Prob. III.

Rule II.—As 360 is to the degrees in the arc of a sector, so is the whole area of the circle, to the area of the sector.

Note. For a semicircle take one half, for a quadrant, one quarter, &c., of the whole circle.

FORMULÆ.

$$A = \frac{1}{2} r \times \text{arc, and } r = \frac{2A}{\text{arc}}.$$

EXAMPLE.

1. What is the area of the sector C A D B, the radius being 10, and the chord A B 16?

By Rule I. .... $100 = AC^2$
$64 = AE^2$

$36(6 = CE$
$10 = CD$

$4 = DE$

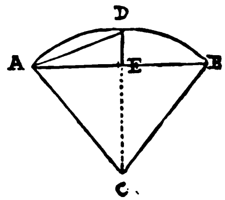
$16 = DE^2$
$64 = AE^2$

$80\ (\ 8{\cdot}9442719 = AD$
8

$71{\cdot}5541752$
16

$3\,)\,55{\cdot}5541752$
$2\,)\,18{\cdot}5180584$ arc ADB
$9{\cdot}2590297 =$ half arc
$10 =$ radius

$92{\cdot}590297$ *Ans.*



2. Required the area of a sector, the arc of which contains 96 degrees, the diameter being 3 feet.

$$\cdot 7854 = \tfrac{1}{4}\pi$$
$$9 = 3^2$$
$$7{\cdot}0686 = \text{area of the whole circle.}$$

Then by Rule II.,
as $360° : 96° :: 7{\cdot}0686$
or, as $30° : 8° :: 7{\cdot}0686 : 1{\cdot}88496$ *square feet. Ans.*

3. What is the area of a sector, the radius of which is 10 feet, and the arc 20? *Ans.* $11\frac{1}{9}$ *square yards.*

4. Required the area of a sector, the radius of which is 18 feet, and the chord of its arc 12? *Ans.* $110\frac{1}{10}$ *square feet.*

5. How many square yards are in a sector of 187° 37′, the radius of the circle being 289? *Ans.* 15194 *square yards.*

6. Required the area of a sector, the radius of which is 25 feet, and its arc contains 147° 29′.

Ans. 804·4 *square feet nearly.*

7. What is the area of a sector, the chord of the arc of which ... height 6? *Ans.* 208.572 *square feet.*

8. Required the area of a sector greater than a semicircle, the chord of its arc being 12, and its diameter 15 feet.

Ans. $124\frac{1}{2}$ *square feet.*

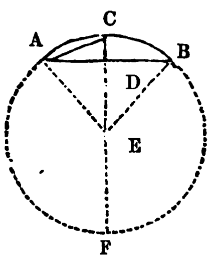
PROBLEM IX.

To find the area of a segment of a circle.

RULE I.—Find the area of a sector having the same arc as the segment, by the last problem; find also the area of the triangle, formed by the chord of the segment and the two radii of the sector: then the difference of these two areas is the area of the segment. See Note 1.

RULE II. Divide the height or versed sine of the segment by the diameter, and find the quotient in the column of versed sines, in Table I., at the end of the book. Take out the corresponding area, in the next column on the right hand, and multiply by the square of the diameter for the area.

FORMULÆ.



Put $r = AE$, $C = AB$, $v = CD$, $p = ED$, $t =$ tabular area, and $a =$ arc ACB; then

$$A = \tfrac{1}{2}(ar - Cp) = \tfrac{2}{3}v\sqrt{C^2 + \tfrac{8}{5}v^2}.$$

$$\tfrac{2}{3}Cv + \frac{v^3}{2C}{}^* = (2r^2)\,t.$$

NOTE 1. When the segment is greater than a semicircle, find the area of the remaining segment, and subtract from the whole area of the circle for the required area.

NOTE 2. The first rule or formula gives an approximate value of the area, not very far from the truth; the second and third are still nearer the truth; and the last rule or formula may be considered as exactly true.

EXAMPLES.

1. Required the area of the segment $ACBDA$, its chord AB being 12, and the radius AE or CE 10 feet.

First find CD and AC from the properties of the figure, and the length of the arc ACB by Prob. VII., Part II.; then find the area by Rule I.; thus $DE = \sqrt{AE^2 - AD^2} = \sqrt{10^2 - 6^2} = 8$, $CD = CE - DE = 10 - 8 = 2$, and $AC = \sqrt{AD^2 + CD^2} = \sqrt{6^2 + 2^2} = 6{\cdot}324555$; whence $\dfrac{6{\cdot}324555 \times 8 - 12}{3} =$ $38{\cdot}59644 =$ arc ACB, and by Rule I., $\frac{1}{2}(38{\cdot}59644 \times 10) - \frac{1}{2}(12 \times 8) = 16{\cdot}3274$ *square feet. Ans.*

* *This formula is due to* B. Gompertz, Esq., F.R.S.

By Rule II. The example being the same as before, we have C D equal to 2; and the diameter 20.

Then 20) 2 (·1
And to ·1 answers .... ·040875 per Table 1.
Square of diameter.... 400

Ans. 16·3500 *square feet.*

By the second formula, the same example being still used, $A = \frac{2}{3} v \sqrt{C^2 + \frac{8}{5} v^2} = \frac{4}{3} \sqrt{12^2 + \frac{8}{5} 2^2} = 16{\cdot}3511$ square feet, which is very near truth.

2. What is the area of the segment, the height of which is 2, and the chord 20 feet. *Ans.* 26·36046.

3. What is the area of the segment, the height of which is 18, and the diameter of the circle 50 feet? *Ans.* 636·375.

4. Required the area of the segment, the chord of which is 16, the diameter being 20 feet. *Ans.* 44·7292.

5. What is the area of a segment, the arc of which is a sextant, the whole circumference of the circle being 25 feet?

Ans. 1·4312 *square feet.*

6. The chord of a segment is 40, and its height 8 feet? what is its area by the third formula? *Ans.* 219·73 *square feet.*

Problem IX.

To find the area of a circular zone.

(*See figure to* Prob. VIII., Part II.)

Rule.—The zone being first divided into a trapezoid (ABCD) and two equal segments (B H D and A C), find the area of the trapezoid by Prob. III., and the areas of the two segments by Prob. IX.; which areas, being added together, will give the area of the zone.

EXAMPLES.

1. The breadth of a zone is 42 feet, and its parallel chords are 48 and 36 feet, required the area.

Ans. 253·08 *square yards.*

2. The two parallel chords of a circular zone are each 100 yards, and the radius of the circle 72 yards; required the area of the zone. *Ans.* 13508½ *square yards.*

3. The parallel chords of a circular zone are each 2½ feet, and -f the circle 1⅔; required the area.

Ans. 6⅐ square *feet nearly.*

Problem X.

To find the area of a circular ring, or space included between two concentric circles.

Take the difference between the two circles, for the ring; or multiply the sum of the radii by their difference, and multiply the product by 3·1416 for the answer.

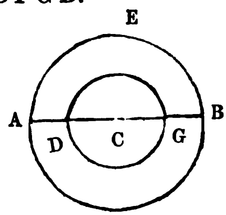
FORMULÆ.

$A = \pi \ (R^2 - r^2) = \frac{1}{4\pi} \ (C^2 - c^2)$; in which R and r are the greater and lesser radii, and C and c the greater and lesser circumferences.

EXAMPLES.

1. The diameters of the two concentric circles being A B 20 and D G 12 feet, required the area of the ring contained between their circumferences A E B A, and D F G D.

A C =	10	3·1416
D C =	6	64
sum	16	12·5664
dif.	4	188·496
	64	201·0624



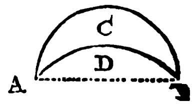
2. The diameters of two concentric circles being 20 and 10 feet; required the area of the ring between their circumferences.
Ans. 235·72 *square feet.*

3. What is the area of a ring, the diameters of its bounding circles being 6 and 4 feet? *Ans.* 15·708.

4. The circular fences on each side a gravel walk, surrounding a shrubbery, are 800 and 714 feet in length; what is the area of the walk, and what did it cost in laying with gravel at 1*s.* $4\frac{1}{2}$*d.* per square yard.
Ans. { *Area* 1151 *square yards.* / *Cost* £79 2*s.* $7\frac{1}{2}$*d.* }

Problem XI.

To find the area of a lune A C B D A.



Rule.—Find by Prob. VIII. the areas of the segments A C B and A D B, formed by the chord A B of the two arcs of the lune, and the difference of these areas will be the area required.

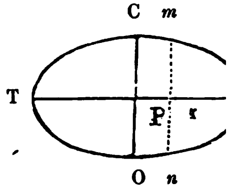
EXAMPLES.

1. What is the area of lune, the chord A B of which is 2 and the heights of its two arcs 5 and $3\frac{1}{2}$ ft.? *Ans.* $25\frac{2}{3}$ *sq.*
2. The chord of a lune is 40 feet, and the heights of its a and 20 feet; required the area. *Ans.* 57·867 *square yar*

Problem XII.

To find the area of an ellipse.

Rule.—Multiply the product of the semiaxes T P, C 3·1416 for the area.



FORMULA.

$A = a\,b\,\pi$, in which a and b are the semiaxes.

EXAMPLES.

1. The axes of an elliptical shrubbery in a park are 300 and 200 feet; required the area.
Ans. 5236 *square yards,* = 1 *acre* 396 *square yar*
2. Required the area of an ellipse, the axes of which a and 50 yards. *Ans.* 2748 *square yards* 8 *fe*

Problem XIII.

To find the area of an elliptical segment, the chord of wh parallel to one of the axes. (See last figure.)

Rule.—Divide the height of the segment by that axis o ellipse of which it is a part; and find in the table of ci segments at the end of the book, a circular segment havin same versed sine as this quotient. Then multiply conti together, this segment, and the two axes, for the area requi

EXAMPLES.

1. What is the area of an elliptic segment m R n, whose h R r is 20; the tranverse T R being 70, and the conjugat 50 feet?

70) 20 ($\cdot285\frac{5}{7}$ the tabular versed sine.
The corresponding segment
is ·185166

$$\begin{array}{r} \cdot185166 \\ 70 \\ \hline 12\cdot961620 \\ 50 \\ \hline 648\cdot081000 \end{array}$$

648·081000 square feet, the area required.

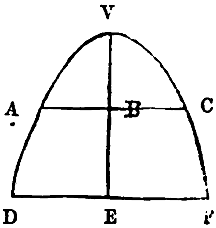
2. What is the area of an elliptic segment, cut off parallel to the shorter axis, the height being 10, and the axes 25 and 35 feet? *Ans.* 162·021 *square feet.*

3. What is the area of the elliptic segment, cut off parallel to the longer axis, the height being 5, and the axes 25 and 35 feet? *Ans.* 97·8458 *square feet.*

PROBLEM XIV.

To find the area of a parabola.

RULE.—Multiply the axis or height VE by the base or double ordinate D F, and $\frac{2}{3}$ of the product will be the area.



FORMULA.

$A = \frac{2}{3} a d$, in which a is the axis, and d the double ordinate.

EXAMPLES.

1. Required the area of the parabola A V C, the axis V B being 2, and the double ordinate A C 12 feet.

$\frac{2}{3} \times 12 \times 2 = 16$ square feet, the area required.

2. The double ordinate of a parabola is 20 feet, and its axis or height 18; required the area of the parabola.

Ans. 240 *square feet.*

PROBLEM XV.

To find the area of a parabolic frustrum A C F D.

Cube each end of the frustrum, and subtract the one cube from the other; then multiply that difference by double the altitude, and divide the product by triple the difference of their squares, for the area.

FORMULA.

$A = \frac{2}{3} a . \frac{C^3 - c^3}{C^2 - c^2}$, in which a is the altitude, and C and c the parallel chords.

EXAMPLES.

1. Required the area of the parabolic frustrum A C F D, A C being 6, D F 10, and the altitude B E 4 feet.

3

Ends.	Squares.	Cubes.
$DF = 10$	100	1000
$AC = 6$	36	216
	64 dif.	784
	3	$8 = 2BE$
	192)	6272 ($32\frac{128}{192} = 32\frac{2}{3}$ *Ans.*
		512
		384
		128

2. What is the area of the parabolic frustrum, the two ends of which are 6 and 10, and its altitude 3 feet. *Ans.* $24\frac{1}{2}$ *square feet.*

NOTE. Those who wish for further information on the areas of the conic sections, are referred to the works of *Emerson*, *Hamilton*, &c., it being foreign to the object of this work to give more on this subject.

PROBLEM XVI.

To find the areas of irregular figures whether bounded by straight lines or curves.

CASE I.—*When the figure is long and narrow.*

RULE.—Take the perpendicular breadth at several places, at equal distances; to half the sum of the first and last two breadths, add the sum of all the intermediate breadths, and multiply the result by the common distance between the breadths for the area.

CASE II.—*When the breadths or perpendiculars are taken at unequal distances, the figure being long and narrow.*

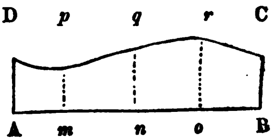
RULE I.—Find the areas of all the trapezoids and triangles separately, and add them together for the area.

RULE II.—Add all the breadths together, and divide the sum by the whole number of them for the mean breadth, which multiply by the length for the area.—This method is not very correct, but may do where great accuracy is not required.

EXAMPLES.

1. The perpendicular breadths, or offsets of an irregular figure at five equidistant places are $AD = 8{\cdot}2$, $mp = 7{\cdot}4$, $nq = 9{\cdot}2$, $or = 10{\cdot}2$, $BC = 8{\cdot}6$ feet; and the common distances $Am =$ &c. $= 50$ feet; required the area.

By Rule I., Case 1.



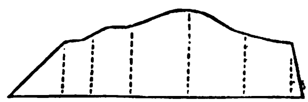
$$\begin{array}{rl}
8{\cdot}2 & \\
8{\cdot}6 & \\
\hline
2)16{\cdot}8 & = \text{sum} \\
\hline
8{\cdot}4 & = \tfrac{1}{2}\ \text{sum} \\
7{\cdot}4 & \\
9{\cdot}2 & \\
10{\cdot}2 & \\
\hline
35{\cdot}2 & \\
50 & \\
\hline
\end{array}$$

Ans. 1760·0 *square feet.*

2. The length of an irregular plank is 25 feet, and its perpendicular breadth at six equidistant places are 17·4, 20·6, 14·2, 16·5, 20·1, and 24·4 inches; required the area.

Ans. $30\frac{1}{8}$ *square feet.*

3. Take the dimensions and find the area of the annexed irregular figure, by Rule I. and II., Case II.

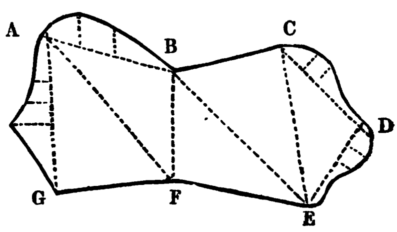


Case III. *When the breadth of the figure is large and its boundary curved or crooked.*

Rule.—Divide the figure into trapeziums and triangles, in the most convenient manner, taking offsets to the curved or crooked portion of the boundary. Find the areas of the trapeziums, triangles, and the offset pieces separately, which, being added together, will give the required area of the whole figure.

EXAMPLE.

The annexed figure is divided into two trapeziums A B F G, B C E F, and one triangle C D E, with offsets on A B, A G, C D, and D E. It is required to measure the several parts of the figure, and to find its area.



The areas of the trapeziums are found by letting fall perpendiculars on the diagonals A F, B E by Prob. IV., and the area of the triangle by Prob. II., the areas of the several offset pieces being found by one or other of the cases of *this Problem.*

PROMISCUOUS EXERCISES.

1. The sides of three squares are 6, 8, and 24 feet; required the side of a square that shall have an area equal to all the three. *Ans.* 26 *feet.*

2. In cutting a circle, the largest possible, out of a card-board 5 feet square, how much will be wasted.
Ans. 5·365 *square feet.*

3. The area of a square is 72 square feet; required the length of its diagonal. *Ans.* 12 *feet.*

4. A ditch 13 yards wide surrounds a circular fortress, the circumference of the fortress being 704 yards; required the area of the ditch. *Ans.* 2 *acres nearly.*

5. What is the area of a circular table the diameter of which is 59 inches. *Ans.* 19 *square feet nearly.*

6. What is the area of an isosceles triangle, the base of which is 5 feet 10 inches, and each side $8\frac{1}{2}$ feet?
Ans. 23 *square feet* $41\frac{1}{3}$ *inches.*

7. Required the side of a decagon the area of which is 9 square feet. *Ans.* 1 *foot* 1 *inch nearly.*

8. The side of a square is 50 yards, and its corners are cut off so as to form an octagon; required the area of the octagon.
Ans. 2071 *square yards.*

PART IV.

MENSURATION OF SOLIDS.

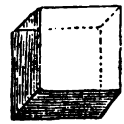
DEFINITIONS.

1. A Solid has three dimensions, length, breadth, and thickness.

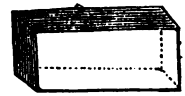
2. A prism is a solid, or body, whose ends are any plane figures, which are parallel, equal, and similar; and its sides are parallelograms.

A prism is called a triangular one when its ends are triangles; a square prism, when its ends are squares; a pentagonal prism, when its ends are pentagons; and so on.

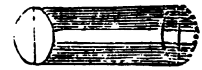
3. A cube is a square prism, having six sides, which are all squares. It is like a die, having its sides perpendicular to one another.



4. A parallelopipedon is a solid having six rectangular sides, every opposite pair of which are equal and parallel.

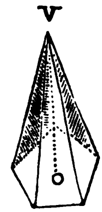


5. A cylinder is a round prism, having circles for its ends.



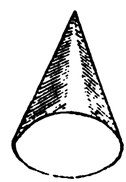
Note. A prism is called a right one, when its sides are perpendicular to its ends; and an oblique prism when its sides are inclined to its ends.

6. A pyramid is a solid having any plane figure for a base, and its sides are triangles, the vertices of which meet in a point at the top, called the vertex of the pyramid.

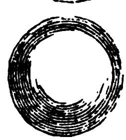


The pyramid takes names according to the figure of its base, like the prism; being triangular, or square, or hexagonal, &c.

7. A cone is a round pyramid, having a circular base.



8. A sphere is a solid bounded by one continued convex surface, every point of which is equally distant from a point within, called the centre.—The sphere may be conceived to be formed by the revolution of a semicircle about its diameter, which remains fixed.



9. The axis of a solid, is a line drawn from the middle of one end, to the middle of the opposite end; as between the opposite ends of a prism. Hence the axis of a pyramid, is the line from the vertex to the middle of the base, or the end on which it is supposed to stand, as O V. And the axis of a sphere, is the same as a diameter, or a line passing through the centre, and terminated by the surface on both sides.

Note. It is called a right pyramid when the axis is perpendicular to the base, but when inclined to the base, it is called an oblique pyramid.

10. The height or altitude of a solid, is a line drawn from its vertex or top, perpendicular to its base.—This is equal to the axis in a right prism or pyramid; but in an oblique one, the height is the perpendicular side of a right-angled triangle, whose hypothenuse is the axis.

11. Also a prism or pyramid is regular or irregular, as its base is a regular or an irregular plane figure.

12. The segment of a pyramid, sphere, or any other solid, is a part cut off the top by a plane parallel to the base of that figure.

13. A frustrum or trunk, is the part that remains at the bottom, after the segment is cut off.

14. A zone of a sphere, is a part intercepted between two parallel planes. When the ends, or planes, are equally distant from the centre, on both sides, the figure is called the middle zone.

15. The sector of a sphere, is composed of a segment less than a hemisphere or half sphere, and of a cone having the same base with the segment, and its vertex in the centre of the sphere.

16. A circular spindle, is a solid generated by the revolution of a segment of a circle about its chord, which remains fixed.

17. A regular body, is a solid contained under a certain number of equal and regular plane figures of the same sort.

18. The faces of the solid are the plane figures under which it is contained; and the linear sides, or edges of the solid, are the sides of the plane faces.

19. There are only five regular bodies: namely, 1st, the tetrahedron, which is a regular pyramid, having four triangular faces: 2nd, the hexahedron, or cube, which has 6 equal square faces: 3rd, the octahedron, which has 8 triangular faces: 4th, the dodecahedron, which has 12 pentagonal faces: 5th, the icosahedron, which has 20 triangular faces.

TABLE OF SOLID MEASURE.

1728 cubic inches	=	1 cubic foot.
[illegible] cubic feet	=	1 cubic yard.
'74, or nearly cubic inches	=	1 gallon.

PROBLEM I.

To find the solidity of a cube.

RULE.—Cube one of its sides for the content; that is, multiply the side by itself, and that product by the side again.

FORMULÆ.

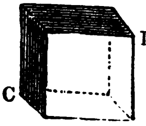
Let l = length of the side of the cube, S its solidity, and s its surface; (*which two last are also used to represent the solidities and surfaces of all the solids in the following problems*) then,

$$S = l^3, \text{ and } l = \sqrt[3]{S}. \quad \text{Also } s = 6\, l^2.$$

EXAMPLES.

1. If the side A B, or A C, or B D, of a cube be 24 inches, what is its solidity or content?

By the Rule or the first Formula.




```
   24
   24
   --
   96
  48
 ----
  576
   24
 ----
 2304
1152
-----
13824 Ans.
```

2. How many solid yards are in a cube the side of which is 22 feet? *Ans.* 394 *solid yards* 10 *feet.*

3. Required how many solid feet are in the cube the side of which is 18 inches? *Ans.* $3\frac{3}{8}$.

4. What is the content of a cube, measuring 6 feet 8 inches every way? *Ans.* 296 *cubic feet* 3′. 6″. 8‴.

5. A cubical box contains 343 cubic feet; required the length of its side.

By the second formula $l = \sqrt[3]{S} = \sqrt[3]{343} = 7$ feet.

6. How many square feet of deal will make a cubical box, lid included, each side of the box being 3 feet?

By the last formula, $s = 6\, l^2 = 6 \times 3^2 = 54$ square feet.

PROBLEM II.

To find the solidity of a parallelopipedon.

RULE.—Multiply the length, breadth, and depth, or altitude,

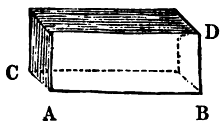
all continually together, for the solid content: that is, multiply the length by the breadth, and that product by the depth.

FORMULÆ.

Put l = length, b = breadth, and d = depth of the solid; then $S = l\,b\,d$, $l = \frac{S}{b\,d}$, $b = \frac{S}{l\,d}$, $d = \frac{S}{l\,b}$, Also, $s = 2\left\{l\,(b + d) + b\,d\right\}$.

EXAMPLES.

1. Required the content of the parallelopipedon, whose length A B is 6 feet, its breadth A C $2\frac{1}{2}$ feet, and altitude B D $1\frac{3}{4}$ feet?



$$\begin{array}{rl} 1{\cdot}75 & = \text{B D} \\ 6 & = \text{A B} \\ \hline 10{\cdot}50 & \\ 2{\cdot}5 & = \text{A C} \\ \hline 5250 & \\ 2100 & \\ \hline 26{\cdot}250 & \textit{Ans.} \end{array}$$

2. Required the content of a parallelopipedon, the length of which is 10·5, breadth 4·2, and height 3·4. *Ans.* 149·94.

3. How many cubic feet are in a block of marble, the length of which is 3 feet 2 inches, breadth 2 feet 8 inches, and depth 2 feet 6 inches? *Ans.* $21\frac{1}{9}$.

4. A stone in the ruins of the walls of Balbec is 36 feet in length, 14 in breadth, and 12 in thickness; required its content, and its weight at the rate of 180 lbs. per cubic foot.

Ans. 11088 *cubic feet, and weight* 891 *tons.*

5. A rectangular cistern is to be made 32 feet in length and 12 in breadth, and to hold 1920 cubic feet of water; what must be its depth?

By the third formula the depth $d = \frac{S}{l\,b} = \frac{1920}{32 \times 12} = 5$ feet.

6. What quantity of deal is there in a box $3\frac{1}{2}$ feet long, 2 wide, and $1\frac{1}{2}$ deep?

By the last formula $s = 2\left\{3\frac{1}{2}\,(2 + 1\frac{1}{2}) + 2 \times 1\frac{1}{2}\right\} = 30\frac{1}{2}$

Problem III.

To find the solidity of any prism or cylinder.

Rule.—Find the area of the base or end; which multiply by the height or length; and the product will be the content.

To find the area of the surface of a prism or cylinder.

Rule.—Multiply the circumference of the base or end by the length or height, and the product will be the area required.

Note. If the *whole* surface be required, the area of the two ends must be added to the area found by the rule.

FORMULÆ.

Put $l =$ length or height, as before; a the area, and c the circumference of the base; then

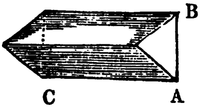
$S = a\,l = \frac{c^2\,l}{4\,\pi}$, $l = \frac{S}{a}$. Also $s = c\,l + 2\,a =$ surface of the prism, including the two ends, and $s = \pi\,r\,l =$ convex surface of the cylinder, exclusive of the ends, r being the radius of the base.

EXAMPLES.

1. Required the content of a triangular prism, the length A C of which is 12 feet, and each side of its equilateral base $2\frac{1}{2}$ feet.

By the Rule or first Formula.

$$\begin{array}{rl} \cdot433013 & \text{tabular No.} \\ 6\frac{1}{4} & = (2\frac{1}{2})^2 \\ \hline 2\cdot598078 & \\ 108253 & \\ \hline a = 2\cdot706331 & \text{area of end} \\ l = \quad\quad 12 & \text{length} \\ \hline \end{array}$$



Ans. 32·475972 *solid feet.*

2. Required the solidity of a triangular prism, the length of which is 10 feet, and the three sides of its triangular end or base, are 5, 4, 3 feet? *Ans.* 60 *cubic feet.*

3. What is the content of a hexagonal prism, the length being 8 feet, and each side of its end 1 foot 6 inches.

Ans. 46·765 *cubic feet.*

4. Required the content of a cylinder, the length of which is 20 feet, and circumference $5\frac{1}{2}$ feet.

By the second formula,

$$S = \frac{c^2\,l}{4\,\pi} = (5\tfrac{1}{2})^2 \times 20 \times \cdot07958 = 48\cdot146 \text{ cubic feet.}$$

5. What is the convex surface of a cylinder, the length of which is 16 feet, and its diameter 2 feet 3 inches?

By the last formula,

$s = \pi\, r\, l = 3{\cdot}1416 \times 2\frac{1}{4} \times 16 = 113{\cdot}0976$ sq. feet.

6. Required the *whole* superficial area of a cylinder, the length of which is 15 feet, and diameter $5\frac{1}{3}$ feet?

Ans. $32\frac{8}{9}$ *square yards.*

7. The *whole* superficial area of a triangular prism is 143 square feet, and each side of its equilateral ends 5 feet; required its length?

By transposing the third formula,

$$l = \frac{s - 2\,a}{c} = 6 \text{ feet } 8 \text{ inches nearly}$$

8. The diameter of a cylinder is 12 feet, and its length 20; required the content?

Ans. 2262 *cubic feet nearly.*

9. How many cubic feet of stone is there in a round pillar, the height of which is 16 feet, and diameter 2 feet 3 in? *Ans.* 63·62 *cubic ft.*

10. How many square yards of painting are there in the convex surface of a column, the length of which is 20 feet, and its diameter 2 feet?

Ans. 13 *square yards* $8\frac{2}{3}$ *feet nearly.*

Problem IV.

To find the solidity of any cone or any pyramid.

Rule.—Compute the area of the base, then multiply that area by the height, and take $\frac{1}{3}$ of the product for the content.

To find the convex surface of a right cone, or the slant surface of a right pyramid.

Rule.—Multiply the circumference of the base by the slant height, or length of the side, and take half the product for the surface.

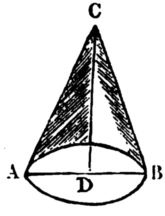
FORMULÆ.

$S = \frac{1}{3}\,a\,l,\ a = \frac{3\,S}{l},\ l = \frac{3\,S}{a}.$ Also $s = \frac{1}{2}\ c\,l'$, l' being the slant height. When the whole surface is required, the area of the base must be added.

EXAMPLES.

1. What is the solidity of a cone, the height C D of which is [illegible] feet, and the diameter A B of the base $2\frac{1}{2}$?

Here $2\frac{1}{2} \times 2\frac{1}{2} \times \frac{5}{2} \times \frac{5}{2} \times \frac{25}{4} = 6\frac{1}{4} = AB^2$.



Then ·7854
$6\frac{1}{4}$

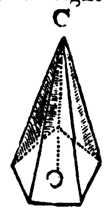
4·7124
19635

4·90875 area of base
$12\frac{1}{2}$ height C D

58·90500
2·454375

3)61·359375
20·453125 *Ans.*

2. What is the solid content of a pentagonal pyramid, its height being 12 feet, and each side of its base 2 feet?



1·720477 tab. area
4 square side

6·881908 area base
4 $\frac{1}{3}$ of height C O

Ans. 27·527632 *cubic feet.*

3. What is the content of a cone, its height being $10\frac{1}{2}$ feet, and the circumference of its base 9 feet?

Ans. 22·561 *cubic feet.*

4, Required the content of a triangular pyramid, its height being 14 feet 6 inches, and three sides of its base, 5, 6, 7.

Ans. 71·0351 *cubic feet.*

5. What is the content of a hexagonal pyramid, the height of which is 6·4, and each side of its base 6 inches.

Ans. 1·38 *cubic feet.*

6. If the diameter of the base A B be 5 feet, and the side of the cone A C 18, required the convex surface.

3·1416
5 diameter

15·7080 circumference
18

125664
15708

2)282·744
Ans. 141.372 *square feet.*

7. What is the convex surface of a cone, the slant side of which is 20, and the circumference of its base 9 feet?

Ans. 90 *square feet.*

8. Required the convex surface of a cone, the slant height of which is 50 feet, and the diameter of its base 8 feet 6 inches?

Ans. 667·59 *square feet.*

9. The side of the equilateral base of a triangular pyramid is 5 feet, and its solid content $62\frac{1}{2}$ cubic feet; required its perpendicular height.

Ans. By the third formula, the height if found 17 *feet* 4 *inches nearly.*

10. Required the weight of a hexagonal pyramid of marble, each side of the base of which is 1 foot 3 inches, and the vertical height 10 feet, the weight of the marble being 170 lbs. per cubic foot. *Ans.* 1 *ton.* 0 *cwt.* $18\frac{1}{2}$ *lbs.*

11. A cone contains 8 solid feet, and its height is 2 feet; what is the circumference of its base? *Ans.* 12·28 *ft nearly.*

12. The circumference of the base of a cone is 33 feet, and the slant height 8 feet 9 inches; required the content?

Ans. 202·65 *cubic feet.*

PROBLEM V.

To find the solidity of the frustrum of a cone, or any pyramid.

GENERAL RULE.—To the area of the two ends add the square root of their product, and multiply the sum by $\frac{1}{3}$ of the height for the solidity.

FORMULÆ.

If A and a be the areas of the greater and lesser ends; then,

$$S = \tfrac{1}{3}\,(A + a + \sqrt{A\,a})\;l.$$

When the solid is the frustrum of a cone, or of a pyramid, having its ends regular polygons.

RULE.—To the sum of the squares of the radii of the ends, if a cone, or of the sides of the ends, if a pyramid, add their product; and multiply the sum by 3·1416, if a cone, or by the tabular number of the polygon, if a pyramid, and again by $\frac{1}{3}$ of the height for the content.

FORMULÆ.

$$S = \tfrac{1}{3}\,(R^2 + r^2 + R\,r)\;l\,\pi,$$

'- which R and r are the radii of the ends, if a cone, or the ᶠ the ends, if a pyramid. In the latter case π represents

the tabular number of the polygon. If R and r be taken as the circumferences of the ends of a cone, then π must be taken $= \cdot 07958$.

To find the convex surface or frustrum of a cone, or the slant surface of a pyramid.

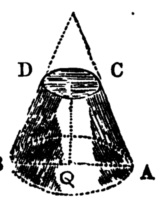
Rule.—Multiply the sum of the circumferences of the two ends by $\frac{1}{2}$ the slant height of the frustrum for the required surface.

Note. When the *whole* surface is required the areas of the two ends must be added to the result of the Rule.

EXAMPLES.

1. What is the content of a frustrum of a cone, the height of which is 20 inches, and the diameters of its two ends 28 and 20 inches?

28	28	20
28	20	20
224	560	400
56	784	
784	400	
	1744	
	·2618	
	13952	
	1744	
	10464	
	3488	
	456·5792	
	20 = P Q	



Ans. 9131·5840 *solid inches.*

2. Required the content of a pentagonal frustrum, the height of which is 5 feet, each side of the base 1 foot 6 inches, an each side of the less end 6 inches.

18	18	6
18	6	6
——	——	—
144	108	36
18	324	—
——	36	
324	——	
——	3) 468	
	——	
	156 $\frac{1}{3}$ of sum.	
	1·720477 tab. area.	
	————	
	10322862	
	8602385	
	1720477	
	————	
	268·394412 mean area	
	5 height PQ	
	————	

$$144\left\}\begin{matrix}12\\12\end{matrix}\right|\begin{matrix}1341\cdot972060\\111\cdot831005\\9\cdot319250\end{matrix}$$ *Ans. in cubic feet.*

3. What is the solidity of the frustrum of a cone, the altitude being 25, the circumference at the greater end 20, and at the less end 10 feet? *Ans.* 464·205 *cubic feet.*

4. How many solid feet are in a piece of timber, whose bases are squares, each side of the greater end being 15 inches, and each side of the less end 6 inches; also the length, or perpendicular altitude is 24 feet? *Ans.* 19$\frac{1}{2}$ *cubic feet.*

5. To find the content of the frustrum of a cone, the altitude being 18, the greatest diameter 8, and the least 4 feet? *Ans.* 527·7888 *cubic feet.*

6. What is the solidity of a hexagonal frustrum, the height being 6 feet, the side of the greater end 18 inches, and of the less 12 inches? *Ans* 24·68 *cubic feet.*

7. The girts of the trunk of a tree at its two ends are 15 and 10 feet, and its length 48 feet; how many solid feet does it contain? (R and r being taken for the girts in the second formula, &c.) *Ans.* 604$\frac{4}{5}$ *nearly.*

The height of the frustrum of an octagonal pyramid is 48 and the sides of its ends 26 and 19; required the solid nt. *Ans.* 118279 *cubic feet.*

9. The sides of the ends of the frustrum of a square pyramid are 6 and 4 feet, and its slant length 20 feet, required its slant surface,

$$\left.\begin{array}{l} 6 \times 4 = 24 \\ 4 \times 4 = 16 \end{array}\right\} \text{circumf. of ends.}$$

$$\begin{array}{r} 40 \text{ sum} \\ 10 = \frac{1}{2} \text{ length} \\ \hline 9)\,400 \\ \hline 44\frac{4}{9} \textit{ square yards.} \end{array}$$

NOTE. The slant length is measured from the middle of one side to that of its corresponding side.

10. The slant height of tower, in the form of a hexagonal pyramid, is 74 feet, each side of the base $7\frac{1}{2}$, each side of the top $2\frac{1}{2}$ feet; required the area of the sides, and the expense of painting it at 1*s.* 3*d.* per square yard.
Ans. 2220 *square feet, and* £15 8*s.* 4*d.*

11. What is the convex surface of the frustrum of a cone, the slant height of the frustrum being 12·5, and the circumferences of the two ends 6 and 8·4 feet? *Ans.* 90 *square feet.*

12. Required the convex surface of the frustrum of a cone, the side of the frustrum being 10 feet 6 inches, and the circumferences of the two ends 2 feet 3 inches, and 5 feet 4 inches?
Ans. $39\frac{13}{16}$ *square feet.*

13. The *perpendicular* height of the frustrum of a cone is 3 feet, and the circumferences of the base and top 9 and 6 feet; required the *whole* surface? *Ans.* 68·35 *square feet.*

PROBLEM VI.

To find the solidity of a wedge.

RULE.—To the length of the edge add twice the length of the back or base, and reserve the sum; multiply the height of the wedge by the breadth of the base; then multiply this product by the reserved sum, and take $\frac{1}{6}$ of the last product for the content.

FORMULA.

$S = \frac{1}{6}(2l + l')\,b\,h$, the symbols denoting the parts shown on the following figure.

EXAMPLES.

1. What is the content in feet of a wedge, the altitude A P of which is 14 inches, its edge A B 21 inches, and the length of its base D E 32 *inches,* and its breadth C D $4\frac{1}{2}$ inches?

21	14
32	$4\frac{1}{2}$
32	—
—	56
85	7
—	—
	63
	85
	—
	315
	504
	—
6	5355
1728 { 12	892·5 *Ans. in cubic inches.*
12	74·375
12	6·197916
	·516493 *Ans. in cubic feet, or little more than half a cubic foot.*

2. The edge and base of a wedge are respectively 9 feet, and 5 feet 4 inches in length, the base is 2 feet 8 inches in breadth, and the height 3 feet 6 in.; required the content of the wedge.

Ans. 30 *cubic feet,* 7′. 1″. 4‴.

3. The height and length of edge, the length and breadth of base of a wedge are each 2 feet; what is its solidity?

Ans. 4 *cubic feet.*

Problem VII.

To find the solidity of a prismoid.

Definition.—The ends of a prismoid are parallel and dissimilar rectangles or trapezoids; the solid is, therefore, the frustrum of a wedge, the part of the wedge next the edge being cut off.

Rule.—Add into one sum, the areas of the two ends and 4 times the middle section parallel to them, and $\frac{1}{6}$ of that sum will be a mean area; which being multiplied by the height, will give the content.

Note. For the length of the middle section, take half the sum of the lengths of the two ends; and for its breadth, take half the sum of the breadths of the two ends.

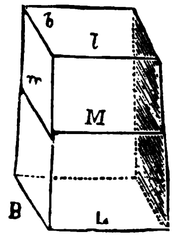
FORMULA.

$S = \frac{1}{6}$ (L B + lb + 4 M m), the symbols representing the parts shown in the following figure.

EXAMPLE.

1. *How many* cubic feet are there in a stone, the ends of

which are rectangles, the length and breadth of the one being 14 and 12 inches; and the corresponding sides of the other 6 and 4 inches: the perpendicular height being $30\frac{1}{2}$ feet?



14	10	6
12	8	4
168	80	24
	4	
	320	
	168	
	24	
	5) 512	

$85\frac{1}{3}$ mean area in inches
$30\frac{1}{2}$ height

2560
$42\frac{2}{3}$

$144 \begin{cases} 12 \\ 12 \end{cases}$ | $2602{\cdot}\dot{6}$
$216{\cdot}\dot{8}$

$18{\cdot}0\dot{7}\dot{4}$ *Ans.*

2. What is the content of a railway coal waggon, of which the length and breadth at top are $81\frac{1}{2}$ and 55 inches at bottom, the length and breadth are 41 and $29\frac{1}{2}$ inches, and the height $47\frac{1}{4}$ inches? *Ans.* $73\frac{1}{9}$ *cubic feet.*

Note. Several railway cuttings are in the form of a prismoid, with dissimilar trapezoidal ends. The following is an example of this kind, the bottom width being the same throughout.

3. The top widths of a railway cutting are 120 and 90 feet, their respective depths 30 and 20 feet, the bottom width 30 feet, and the length of the cutting 3 chains or 66 yards; required the content in cubic yards. *Ans.* $12466\frac{2}{3}$ *cubic yards.*

Problem VIII.

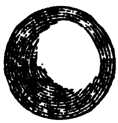
To find the solidity of a sphere or globe.

Rule.—Multiply the cube of the diameter by ·5236.

FORMULÆ.

$$S = \tfrac{1}{6} D^3 \pi, \text{ and } D = \sqrt[3]{\frac{6\,S}{\pi}}$$

EXAMPLES.



1. The diameter of a sphere is 12 feet required its solidity?

$12^3 \times \cdot5236 = 904\cdot7808$ *cubic feet.*

2. Find the content and weight of an ivory ball $3\frac{1}{2}$ inches in diameter, the weight of ivory being 1820 ounces (Av.) per cubic foot.

Ans. Content 22·448 *cubic in.*, and weight 24 *ounces nearly.*

3. A $2\frac{1}{2}$ inch cube of ivory is turned into a sphere of the same diameter; what weight of ivory will be lost?

Ans. 7·68 *ounces.*

4. Required the solid content of the earth, supposing its circumference to be 25000 miles?

Ans. 263858149120 *cubic miles.*

Problem IX.

To find the solidity of a spherical segment.

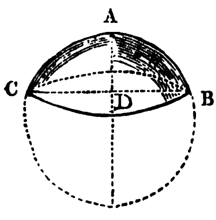
Rule.—To three times the square of the radius of its base, add the square of its height; then multiply the sum by the height, and the product again by ·5236.

FORMULA.

$S = \frac{1}{6}(3\,r^2 + h^2)\,h\,\pi$, in which $r = AB$, and $h = CD$.

EXAMPLES.

1. Required the content of a spherical segment, its height being 4 inches, and the radius of its base 8?



8	4	·5236
8	4	832
64	16	10472
3	192	15708
		41888
192	208	
	4	435·6352 *Ans.*
	832	

2. What is the solidity of the segment of a sphere, the height of which is 9, and the diameter of its base 20 feet?

Ans. 1795·4244 *cubic feet.*

3. Required the content of the spherical segment, the height of which is $2\frac{1}{4}$, and the diameter of its base 8·61684 feet?

Ans. 71·5695 *cubic feet.*

PROBLEM X.

To find the solidity of a spherical zone or frustrum.

RULE.—Add together the square of the radius of each end, and $\frac{1}{3}$ of the square of their distance, or of the height; then multiply the sum by the said height, and the product again by 1·5708.

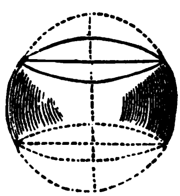
FORMULA.

$$S = \frac{1}{6} \{ 3 (R^2 + r^2) + h^2 \} h \pi.$$

EXAMPLES.

1. What is the solid content of a zone, its greater diameter being 12 inches, the lesser 8, and the height 10 inches?

$$\begin{aligned} 6^2 &= 36 \\ 4^2 &= 16 \\ \tfrac{1}{3} \times 10^2 &= 33\tfrac{1}{3} \\ \hline & \ 85\tfrac{1}{3} \end{aligned}$$



$85\frac{1}{3} \times 10 \times 1·5708 = 1340·416$ *cubic in.*, the content required.

2. Required the content of a zone, the greater diameter is 12, less diameter 10, and height 2 feet. *Ans.* 195·8264 *cubic ft.*

3. What is the content of a middle zone, the height being 8 feet, and the diameter of each end 6 feet?

Ans. 494·2784 *cubic ft.*

4. A cask is in the form of the middle zone of a sphere, its top and bottom diameters being 5 feet 8 inches, and its height 5 feet, inside measure; how many gallons will it contain?

Ans. $1193\frac{3}{4}$ *gallons.*

PROBLEM XI.

To find the convex surface of a sphere, also of a segment and zone thereof.

For the sphere.

RULE.—Multiply the square of the diameter by 3·1416.

For the segment or zone.

RULE.—Multiply the circumference of the whole sphere by the height of the segment or zone.

and from floor to floor, as far as they extend, for the other; then multiply the length by the height.

In measuring joiners' work, the string is made to ply close to every part of the work over which it passes.

In roofing, the length of the rafters is equal to the length of a string stretched from the ridge down the rafter, and along the eaves-board, till it meets with the top of the wall. This length multiplied by the common depth and breadth of the rafters, gives the content of one, and that multiplied by the number of them, gives the content of all the rafters.

King post roofs, &c., all the timbers in a roof are measured in the same manner as the joists, &c., in flooring. In the annexed figure, representing a truss for a roof, all the beams, as the tie-beam, king-post, braces, &c., are measured to their full lengths, breadths, and thicknesses, including the lengths of tenons; also the parts cut out on each side of the king-post, to form abutments for the braces, are included; unless their lengths exceed 2 feet each by 3 inches breadth, when their solidities must be deducted, pieces of smaller size, being considered of little or no value, are, therefore, included in the measurement.

For stair-cases, take the breadth of all the steps, by making a line ply close over them, from the top to the bottom; and multiply the length of this line by the length of a step for the whole area.—By the length of a step, is meant the length of the front and the returns at the two ends; and by the breadth, is to be understood the girt of its two outer surfaces, or the tread and rise.

For the balustrade, take the whole length of the upper part of the hand-rail, and girt over its end till it meet the top of the newel post, for one dimension; and twice the length of the baluster upon the landing, with the girt of the hand-rail, for the other dimension.

For wainscoting, take compass of the room for one dimension; and the height from the floor to the ceiling, making the string ply close into all the mouldings, for the other dimension.—Out of this must be made deductions, for windows, doors, and chimneys, &c.

For doors, it is usual to allow for their thickness, by adding *it into both* the dimensions of length, and breadth, and then

multiply them together for the area.—If the door be pannelled on both sides, take double its measure for the workmanship: but if one side only be pannelled, take the area and its half for the workmanship.—*For the surrounding architrave,* girt it about the outermost part for one dimension, and measure over it as far as it can be seen when the door is open, for the other.

Window-shutters, bases, &c., are measured in the same manner.

EXAMPLES.

1. Required the content of a floor 48 feet 6 inches long, and 24 feet 3 inches broad.

Decimals.	Duodecimals.
48·5	48 6
$24\frac{1}{4}$	24 3
1940	204 0
970	96
12·125	12 1 6
1176·125 feet	1176 1 6
11·76125 squares	*Ans.* 11·76 1 6

2. A floor being 36 feet 3 inches long, and 16 feet 6 inches broad, how many squares are in it? *Ans.* 5 *squares* $98\frac{1}{8}$ *feet.*

3. How many squares are there in 173 feet 10 inches in length, and 10 feet 7 inches height, of partitioning?

Ans. 18·3972 *squares.*

4. What cost the roofing of a house at 10*s.* 6*d.* a square; the length, within the walls, being 52 feet 8 inches, and the breadth 30 feet 6 inches: reckoning the roof $\frac{3}{2}$ of the flat?

Ans. £12 12*s.* $11\frac{3}{4}$*d.*

5. To how much, at 6*s.* per square yard, amounts the wainscoting, of a room; the height, taking in the cornice and mouldings being 12 feet 6 inches, and the whole compass 83 feet 8 inches; also the three window shutters are each 7 feet 8 inches by 3 feet 6 inches, and the door 7 feet by 3 feet 6 inches; the door and shutters, being worked on both sides, are reckoned work and half work? *Ans.* £36 12*s.* $2\frac{1}{2}$*d.*

6. In a naked floor there are 2 girders, each 20 feet long, and 1 foot 2 inches by 1 foot; there are 16 bridging joists, each 20 feet long, and $6\frac{1}{2}$ inches by 3; 16 binding joists, each 9 feet in length, and $8\frac{1}{2}$ inches by 4: 48 ceiling joists, each 6 feet long, and 4 inches by $2\frac{1}{2}$: required the content in cubic feet.

Ans. 144 *cubic feet.*

7. What will the wainscoting of a room cost at 4*s*. per square yard; the height of the room, including cornice and the mouldings, is $12\frac{1}{2}$ feet and the compass $125\frac{1}{2}$ feet; there are three window shutters, each 7 feet 8 inches by $3\frac{1}{2}$ feet, and the door 7 feet by $3\frac{1}{2}$ feet; the door and shutters, being worked on both sides, are reckoned half work additional? *Ans.* £36 12*s*. $2\frac{1}{2}$*d*.

SLATERS' AND TILERS' WORK.

In these articles, the content of a roof is found by multiplying the length of the ridge by the girt over from eaves to eaves; making allowance in this girt for the double row of slates at the bottom, or for how much one row of slates or tiles is laid over another.

When the roof is of a true pitch, that is, forming a right angle at top; then the breadth of the building with its half added, is the girt over both sides.

In angles formed in a roof, running from the ridge to the eaves, when the angle bends inwards, it is called a valley; but when outwards, it is called a hip.

Deductions are made for chimney shafts or window holes.

EXAMPLES.

1. Required the content of a slated roof, the length being 45 feet 9 inches, and the whole girt 34 feet 3 inches.

Decimals.	Duodecimals.
45·75	45 9
$34\frac{1}{4}$	34 3
18300	205 6
13725	135
114375	11 5 3
9)1566·9375 feet	9)1566 11 3
yards 174·104	174 *yds.* 11′ 3″.

2. To how much amounts the tiling of a house, at 25*s*. 6*d*. per square; the length being 43 feet 10 inches, and the breadth on the flat 27 feet 5 inches, also the eaves projecting 16 inches on each side, and the roof of a true pitch? *Ans.* £24 9*s*. $5\frac{1}{2}$*d*.

PLASTERERS' WORK.

erers' work is of two kinds, namely, ceiling, which is ng upon laths; and rendering, which is plastering upon which are measured separately.

:ontents are estimated either by the foot or yard, or square feet. Enriched mouldings, &c., are rated by running or easure.

ctions are to be made for chimneys, doors, windows, &c.

EXAMPLES.

ow many yards contain the ceiling, which is 43 feet 3 ong, and 25 feet 6 inches broad?

Decimals.	Duodecimals.
43·25	43 3
$25\frac{1}{2}$	25 6
21625	221 3
8650	86
21625	21 7 6
9) 1102·875	9) 1102 10 6
'ards 122·541	*Ans.* 122 *yds.* 4 *ft.* 10′ 6″.

› how much amounts the ceiling of a room, at 10*d.* per he length being 21 feet 8 inches, and the breadth 14 feet :s? *Ans.* £1 9*s.* $8\frac{3}{4}$*d.*

ıe length of a room is 18 feet 6 inches, the breadth 12 ches, and height 10 feet 6 inches; to how much amounts ng and rendering, the former at 8*d.* and the latter at 3*d.* l; allowing for the door of 7 feet by 3 feet 8, and a fire- 5 feet square? *Ans.* £1 13*s.* 3*d.*

:quired the quantity of plastering in a room the length l feet 5 inches, breadth 13 feet 2 inches, and height 9 feet to the under side of the cornice, which girts $8\frac{1}{2}$ inches, ects 5 inches from the wall on the upper part next the deducting only for a door 7 feet by 4.

Ans. 53 *yds.* 5 *ft.* 3 *in.* *of rendering.*
18 5 6 *of ceiling.*
39 $0\frac{11}{12}$ *of cornice.*

PAINTERS' WORK.

Painters' work is computed in square yards. Every part is measured where the colour lies; and the measuring line is forced into all the mouldings and corners.

Windows are done at so much a piece; and it is usual to allow double measure for carved mouldings, &c.

EXAMPLES.

1. How many yards of painting contains the room which is 65 feet 6 inches in compass, and 12 feet 4 inches high?

Ans. 89 *yards* 6 *feet* 10′.

2. The length of a room being 20 feet, its breadth 14 feet 6 inches, and height 10 feet 4 inches; how many yards of painting are in it, deducting a fire-place of 4 feet by 4 feet 4 inches, and two windows each 6 feet by 3 feet 2 inches? *Ans.* $73\frac{2}{27}$ *yds.*

3. What cost the painting of a room at 6*d.* per yard; its length being 24 feet 6 inches, its breadth 16 feet 3 inches, and height 12 feet 9 inches; also the door is 7 feet by 3 feet 6, and the window shutters to two windows each 7 feet 9 by 3 feet 6, but the breaks of the windows themselves are 8 feet 6 inches high, and 1 foot 3 inches deep: deducting the fire-place of 5 feet by 5 feet 6? *Ans.* £3 3*s.* $10\frac{1}{2}$*d.*

GLAZIERS' WORK.

Glaziers take their dimensions either in feet, inches, and parts, or feet, tenths and hundredths. And they compute their work in square feet.

In taking the length and breadth of a window, the cross bars between the squares are included. Also windows of round or oval forms are measured as square, measuring them to their greatest length and breadth, on account of the waste in cutting the glass.

EXAMPLES.

1. How many square feet contains the window which is 4·25 feet long, and 2·75 feet broad?

Decimals.		Duodecimals.		
2·75		2	9	
$4\frac{1}{4}$		4	3	
11·00		11	0	
·6875			8	3
11·6875	*Ans.*	11	8	3

2. What will the glazing a triangular sky-light come to at 10*d*. per foot; the base being 12 feet 6 inches, and the perpendicular height 6 feet 9 inches? *Ans*. £1 15*s*. 1$\frac{3}{4}$*d*.

3. There is a house with three tier of windows, three windows in each tier, their common breadth 3 feet 11 inches;

now the height of the first tier is 7 feet 10 inches,
of the second 6 8
of the third 5 4

Required the expense of glazing at 14*d*. per foot.
Ans. £13 11*s*. 10$\frac{1}{2}$*d*.

4. Required the expense of glazing the windows of a house at 13*d*. a foot; there being three stories, and three windows in each story: the height of the lower tier is 7 feet 9 inches,
of the middle 6 6
of the upper 5 3
and of an oval window over the door 1 10$\frac{1}{2}$

The common breadth of all the windows being 3 feet 9 inches.
Ans. £12 5*s*. 6*d*.

PAVIORS' WORK.

Paviors' work is done by the square yard, and the content is found by multiplying the length by the breadth.

EXAMPLES.

1. What cost the paving a foot-path at 3*s*. 4*d*. per yard; the length being 35 feet 4 inches, and breadth 3 feet 3 inches?
Ans. Content 32 *yards* 3 *feet* 6′. Cost £5 7*s*. 11$\frac{1}{2}$*d*.

2. What will be the expense of paving a rectangular court yard, whose length is 63 feet, and breadth 45 feet; in which there is laid a foot path of 5 feet 3 inches broad, running the whole length, with broad stones, at 3*s*. a yard; the rest being paved with pebbles at 2*s*. 6*d*. a yard? *Ans*. £40 5*s*. 10$\frac{1}{2}$*d*.

PLUMBERS' WORK.

Plumbers' work is rated at so much a pound, or else by the hundred weight of 112 pounds.

Sheet lead used in roofing, guttering, &c., is from 7 to 12 lbs. to the square foot. And a pipe of an inch bore is commonly 13 or 14 lbs. to the yard in length.

EXAMPLES.

1. How much weighs the lead which is 39 feet 6 inches long, and 3 feet 3 inches broad, at $8\frac{1}{2}$ lbs. to the square foot?

Decimals.		Duodecimals.		
39·5		39	6	
$3\frac{1}{4}$		3	3	
118·5		118	6	
9·875		9	10	6
128·375		128	4	6
$8\frac{1}{2}$		$8\frac{1}{2}$		
		1024		
1027·000		64		
64·1875		$2\frac{5}{8}$		
		$0\frac{11}{48}$		
1091·1872	*Ans.*	$1091\frac{9}{48}$ *lbs.*		

2. What cost the covering and guttering a roof with lead, at 18*s.* the cwt.; the length of the roof being 43 feet, and breadth or girt over it 32 feet; the guttering 57 feet long, and 2 feet wide: the former 9·831 lbs., and the latter 7·373 lbs. to the square foot? *Ans.* £115 9*s.* $1\frac{1}{2}$*d.*

ARCHED AND VAULTED ROOFS.

To find the concave surface of a circular, gothic, or elliptical vaulted roof.

RULE.—Multiply the length of the arch by the length of the vault for the required surface.

NOTE.—The length of the arch, for most practical purposes, is found by applying a small chord along its concavity, and then measuring its length.

EXAMPLE.

What is the concave surface of Caerfylly bridge, over the Taafe; which is a segment of a circle, span 140, height 35, and width 12 feet? *Ans.* 1944·4 *square feet.*

To find the content of a circular, gothic, or elliptical roof.

RULE.—Multiply the area of the end by the length of the roof for the content of the vacuity.

To find the solid content of the materials.

From the solid content of the whole arch take that of the vacuity for the solid content of the materials.

EXAMPLE.

Required the solidity of the vacuity and of the materials of a

rcular vault; span 36 feet, height 18 feet, thickness of walls the spring 6 feet, thickness of crown 4 feet, and length of the ult 100 feet?

Ans. { 1884·96 *cubic yards solidity of vacuity.*
2026·15 *cubic yards solidity of materials.*

To find the surface and solidity of a dome, the height and imensions of the base being given.

Rule.—Take twice the area for the surface, and multiply the ea of the base by $\frac{2}{3}$rd of the height.

Note. Although these rules are only true when the domes are hemi-herical, yet they are sufficiently near the truth for all practical purposes.

EXAMPLE.

Required the surface and solidity of a hemispherical dome, the ameter of its base being 60 feet.

Ans. { Surface 314·16 *square yards.*
Solidity 2094·4 *cubic yards.*

Note 1. The surface of a saloon is found in the same manner as a vaulted of.

Note 2. Rules might have been here given for the measurement of hay-acks, coal-heaps, &c.; but these may be readily resolved into two or more those solids, the methods of finding the content of which are given in the ensuration of Solids, Part IV. Moreover, haystacks are usually sold by eight, and seldom or never by measurement.

SPECIFIC GRAVITY.

The specific gravity of bodies are their weights when com-ared with an equal bulk of pure water, which, at the tempera-ure of 40°, weighs 1000 ounces avoirdupois per cubic foot. The llowing table, therefore, contains the weights of a cubic foot of veral bodies in ounces.

A TABLE OF THE SPECIFIC GRAVITY OF BODIES.

Platinum	21470	Light earth	1984
Gold	19260	Solid Gunpowder	1745
Mercury	13600	Sand	1520
Lead	11325	Coal from 1030 to	1300
Silver	10470	Pitch	1150
Copper	9000	Box-wood	1030
Cast brass	8400	Sea-water	1030
Steel	7850	Common water	1000
Iron	7704	Mahogany	1065
Cast Iron	7065	Oak	925
Tin	7320	Ash	755
Granite	3950	Beech	700
Flint Glass	3000	Elm	600
Marble	2700	Fir	540
Freestone	2500	Cork	240
Clay	2160	Air	$1\frac{1}{4}$
Brick	2000		

To find the weight of a body from its bulk.

RULE.—Multiply the content of the body, in cubic feet, by its tabular specific gravity for its weight in avoirdupois ounces.

EXAMPLES.

1. Required the weight of a block of marble, whose length is 63 feet, and breadth and thickness each 12 feet, being the dimensions of one of the stones in the walls of Balbec.

$\frac{63 \times 12^2 \times 2700}{16 \times 112 \times 20} = 683\frac{7}{16}$ tons, which is equal to the burden of a second rate East India ship.

2. What is the weight of a block of dry oak, which measures 10 feet long, and 3 feet by $2\frac{1}{2}$? *Ans.* 4336 *lb. nearly.*

To find the magnitude of a body from its weight.

RULE.—Divide its weight in avoirdupois ounces by its tabular specific gravity for its content in cubic feet.

EXAMPLES.

1. Required the content of an irregular block of freestone, which weighs 1 cwt.

$$\frac{112 \times 16}{2520} = \text{cubic feet,}$$

$$\text{and } \frac{112 \times 16 \times 1728}{2520} = 1228{\cdot}8 \text{ cubic inches.}$$

2. How many cubic feet are there in a ton of dry oak?
Ans. $38\frac{138}{185}$ *cubic feet.*

3. A cast iron pipe is 6 inches diameter in the bore and 1 inch in thickness; required the weight of a running foot.
Ans. 67·45 *lbs.*

LAND SURVEYING.

DESCRIPTION OF INSTRUMENTS USED FOR MEASURING AND PLANNING SURVEYS.

THE CHAIN.

THE chain, usually called Gunter's chain, is almost generally used in the British dominions, for measuring the distances required in a survey. It is 66 feet, or 4 poles, in length, and is divided into 100 links, which are joined by rings. The length *of each link,* together with half the rings connecting it with the

adjoining links, is consequently $\frac{66}{100}$ of a foot, or $\frac{66\times 12}{100}=7{\cdot}92$ inches. At every tenth link from each end is attached a piece of brass with notches; that at the tenth link has one notch, that at the 20th two notches, that at the 30th three, that at the 40th four, the middle of the chain, or the 50th link being marked with a large round piece of brass; hence, any distance on the chain may be readily counted. Part of the first link, at each end, is formed into a large ring for the purpose of holding it with the hand.

The chain acquires extension by much use, it should therefore, be frequently examined, and adjusted to the proper length by taking out some of the rings between the links: for this purpose, chains having three rings between each link are to be preferred to those having only two.

THE OFFSET STAFF.

The offset staff is used to measure short distances, called offsets; hence its name. It is usually ten links in length, the links being numbered thereon with the figs. 1, 2, 3, &c. It is usually pointed with iron at one end, for the purpose of fixing it in the ground, as an object for ranging lines, for marking stations, &c.

THE CROSS.

The cross is an instrument used by surveyors to erect perpendiculars. It is usually a round piece of sycamore, box, or mahogany, about four inches in diameter, with two folding sights at right angles to each other, or more commonly with two fine grooves sawed at right angles to each other, which answer the purpose of sights. It is sometimes fixed on a staff of convenient length for use, pointed with iron at the bottom, that it may be fixed firmly in the ground: but it is found more commodious in practice to have a small pocket cross, which may be readily fitted to the offset-staff, either by an iron spike on the cross being inserted in a hole made in the offset-staff, or the offset-staff being passed through a hole made in the cross, to about the eighth link from the piked end, at which place the staff must be shouldered, that the cross may rest firmly.

DIRECTIONS FOR MEASURING LINES ON THE GROUND.

Besides the instruments already described, ten arrows must be provided, about 12 inches long, pointed at the end, so as to be readily pressed into the ground, and turned at the other end, so as to form a *ring to* serve for a handle.

In using the chain, marks are to be set up at the extremities of the line to be measured, as well as its intermediate points, if its extremities cannot be seen from one another, on account of hills, woods, hedges, or other obstructions. Two persons are then required by the surveyor to perform the measurement. The chain leader starts with the ten arrows in his left hand, and one end of the chain in his right; while the follower remains at the starting point, who, looking at the staff or staves, that mark the line to be measured, directs the leader to extend the chain in the direction of the staff or staves. The leader then puts down one of his arrows, and proceeds a second chain's length in the same direction, while the follower comes up to the arrow first put down. A second arrow being now put down by the leader, the first is taken up by the follower; and the same operation is repeated till the leader has expended all his arrows. Ten chains, or 1000 links, having now been measured and noted in the field book, the follower returns the ten arrows to the leader, and the same operation is repeated as often as necessary. When the leader arrives at the end of the line, the number of arrows in the follower's hand shows the number of chains measured since the last exchange of arrows noted in the field book, and the number of links extending from the last arrow to the mark or staff at the extremity of the line, being also added, gives the entire measurement of the line. Thus, if the arrows have been exchanged seven times, and if the follower have six arrows, and from the arrow last put down to the end of the line be 83 links, the whole measurement will be 7683 links, or 76 chains 83 links, which is usually written thus—76·83 chains, the two last figures being decimals of a chain.

$$\begin{array}{r} 7000 \\ 600 \\ 83 \\ \hline 7683 \end{array}$$

In using the chain, care must be taken to stretch it always with the same tension, as it will extend by much use, and will therefore require to be examined occasionally, and shortened, if necessary. But a good chain may be used several days on tolerably smooth ground, without any material extension.

The surveyor must mark, or caused to be marked, every station on the line, while it is being measured, with a staff or cross on the ground, entering its distance in the field book.

When a survey is made for a finished plan, all remarkable objects should be noted down; as buildings, roads, rivers, ponds, footpaths, gates, &c.

The boundary of the estate measured ought to be carefully *observed.* If the ditch be outside the boundary fence, it usually

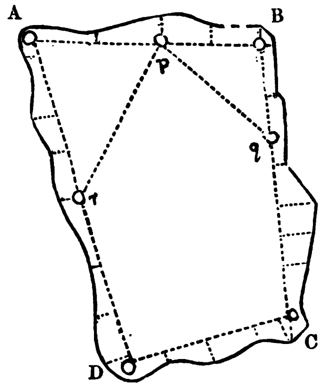
Problem II.

FOUR SIDED FIELDS.

When a field has four sides, straight or crooked, measure the four sides, or lines near them, if crooked, taking the offsets: also measure one or both the diagonals, one of which will serve as a base in plotting the work, and the other for a proof-line; or the proof-line may be measured in any other direction that may be most convenient.

Sometimes the measurement of both the diagonals is prevented by obstructions, in such cases it will be sufficient to measure tie-lines across two of the angles of the trapezium, at the distance of from two to five chains from each angle, according to the size of the field. These tie lines with their distances from the angles on the main lines will be found sufficient for planning the lines and proving them.

EXAMPLE.



In the annexed figure the lines A B, B C, C D, D A are measured, marks being left at p, q, and r, and their respective distances on the lines noted in the field book, thus furnishing the following method of laying down the plan.

On A B, as a base, take A p = given distance, and with the distances A r, p r, and centres A and p describe arcs cutting in r; then prolong A r, and lay off thereon the given length A D. In the same manner construct the triangle p B q, and make B C = its given length. Lastly, join D C, which must be of the length shewn in the field book, otherwise there has been some mistake either in the measurement, or in laying it down. Should this be the case, the whole of the work, firstly on the plan, and secondly in the field, must be gone over again till the error be discovered.

Problem III.

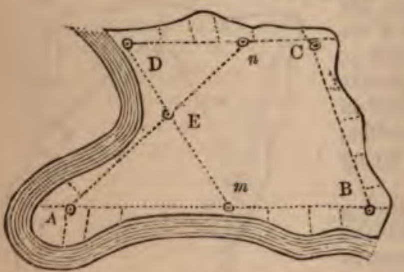
FIELDS HAVING MORE THAN FOUR SIDES.

Various methods will suggest themselves to the surveyor for taking lines to lay down a field that requires more than four main lines to take its boundary. The method of dividing such

fields into trapeziums and triangles is, in most cases, circuitous, and displays little skill on the part of the surveyor, especially where all the sides are crooked, and where a plan is required. A few methods of surveying fields of this kind will, therefore, be presented to direct the student; although their variety of shape is so endless, that no general rule can be given for laying out lines on the ground, that shall give an incontestably accurate plan. To tie every angle in succession, though true in principle, is by no means a safe method, especially where there are a great number of angles to be tied, as an error in one of the tie-lines will derange the whole of the work, without affording the means of detecting where the error lies.

NOTE. The following examples of surveys of this kind occurred in part of the author's extensive practice, as a surveyor of parishes under the Tithe-Commissions. The student is recommended to sketch the following specimens on a large scale, and find their contents by the usual methods.

EXAMPLE.



Here a field of five sides is surveyed by the same number of lines, viz. A B, B C, C D, D m, and A n, the last two intersecting in E. These lines evidently constitute a decisive proof among themselves, and all of them are available in taking the boundary.

In surveying this field (poles or natural marks being supposed to be fixed at A, B, C, D, and E) commence close to the river's edge, in the line A B prolonged backwards, enter the offsets and the station A in the field-book. On arriving at ⊙ m, in the direction E D, enter its distance, and so on to ⊙ B, measuring the line to the fence; from B proceed to C, in like manner, measuring beyond the station to the fence. The place of the ⊙ n is to be noted, on arriving in the direction E A, while measuring C D. D m is next measured, the place of the ⊙ E being noted. Lastly, go from m to A, and measure A n, entering the place of the ⊙ E a second time, all the offsets being supposed to be taken during the operation.

Construction of the plan. Select the distances A m, A E, and E m from the field-book, and with them construct the triangle A m E, prolong the sides to their entire lengths, up to the

of reference, and marked thus ⊙. The bearing of the first main line is usually taken by surveyors, from which the position of the plan with respect to the north is determined. This may be done by a common pocket compass, where great accuracy is not required.

R. of ⊙ 2, and L. of ⊙ 5, &c., denote that the following lines are measured to the right of station 2, and to the left of station 5, respectively.

To Survey with the Chain and Cross.

An acre of land is equal to 10 square chains, that is 10 chains in length and one in breadth, or 1000 links in length and 100 in breadth; an acre, therefore, contains 100,000 square links, as per table of square measure below. Hence the contents in square links are, in the following examples, divided by 100,000, or what is the same thing 5 figures to the right are cut off for decimals, the figures remaining on the left being acres. The decimals are then multiplied by 4 for rods, and again by 40 for poles.

The following tables exhibit the number of chains and links in the different units of lineal measure, and the number of square chains and links in the different units of square measure.

A TABLE OF LINEAR MEASURE.

Links.	Feet.	Yards.	Poles.	Chains.	Furlongs.	Mile.
25	16½	5½	1			
100	66	22	4	1		
1,000	660	220	40	10	1	
8,000	5,280	1,760	320	80	8	1

A TABLE OF SQUARE MEASURE.

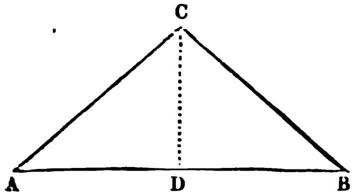
Sq. Links.	Sq. Feet.	Sq. Yards.	Sq. Poles or Perches.	Sq.Chs.	Rods.	Acres.	Sq. Mile.
625	272¼	30¼	1				
10,000	4,356	484	16	1			
25,000	10,890	1,210	40	2½	1		
100,000	43,560	4,840	160	0	4	1	
64,000,000	27,878,400	3,097,600	102,400	6,400	2,560	640	1

Problem I.

TRIANGULAR FIELDS.

1. Let A B C be a triangle, of which the survey, plan, and content are required.

Set up poles or marks at the angles A, B, and C, and measure from A towards B, and when at or near D, try with the cross for the place of the perpendicular C D; plant the cross and turn it till the marks A and B can be seen through one of the grooves; then look through the other groove, and if the mark at C can be seen through it, the cross is in the right place for the perpendicular; if not, move the cross backward or forward till the three marks can be seen as before directed. Suppose the distance A D to be 625 links, and the whole A B, to be 1257 links; return to D, and measure the perpendicular D C, which suppose to be 628 links, thus completing the survey of the triangle.



CONSTRUCTION.

From a scale of equal parts, or plotting scale, lay off the base A B = 1257 links; on which take A D = 625 links; at D erect the perpendicular D C, which make = 628 links; join A C, C B, then A B C is the plan of the triangle.

TO FIND THE CONTENT.

Rule.—Multiply the base by the perpendicular, and half the product will be the area.

EXAMPLES.

1. The dimensions being the same as found above, required the content.

Ans. 1257 × 628 ÷ 2 = 3·94698 acres = 3*a*. 3*r*. 32*p*.

2. The distance from the beginning of the base to the place of the perpendicular is 375 links, the whole base 954, and the perpendicular 246: what is the area of the triangle.

954 × 246 ÷ 2 = 1·17342 = 1*a*. 0*r*. 27½*p*. the content.

3. Measuring the base of a triangle the place of the perpendicular was found at 863 links, and its length 645; the whole base was 1434 links; required the plan and area.

Area. 4*a*. 2*r*. 20*p*.

Problem II.

FIELDS IN THE FORM OF TRAPEZIUMS.

Fields in this form are usually divided into two triangles by iagonal, which is a base to both the triangles.

Let A B C D be a field in the form of a trapezium, the plan l area of which is required.

Measure from A towards C; and let the place of the perpen-ular *m* B be at 5·52, and its length 3·76, also let the place of perpendicular *n* D be at 11·82, and its length 3·44, and the gth of the whole diagonal A C be 13·91 chains, which com-tes the survey: but it is usual also to measure the other dia-ıal B D for a proof line, which is found to be 9·56 chains.

Note 1. The construction of each of the two triangles, forming the trape-n, is the same as the construction given to the first example in Prob. I.

Note 2. The longer of the two diagonals should always be selected for the of the two triangles forming the trapezium, for sometimes the perpen-lar will not fall on the shorter diagonal, without its being prolonged; and n this is the case with both diagonals, one of the sides may be taken for se, or two of the sides, if necessary.

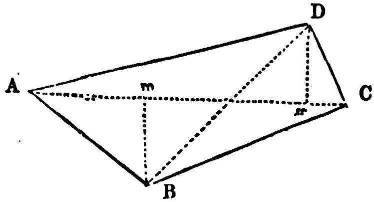
TO FIND THE CONTENT.

Rule.—Multiply the sum of the two perpendiculars by the gonal, and half the product will be the content.

EXAMPLES.

1. Let the measurement of a trapezium be as above found; uired the content.

344
376
———
720
1391
———
27820
9737
———
2) 10·01520
———
5·00760
4
———
0·03040
40
———
1·21600



Ans. 5a. 0r. 1p.

2. From the following notes, plan and find the content of a field.

Perpendiculars on left.	Base or Station Line.	Perpendiculars on right.
	to ⊙ C	
	3250	
	2504	1046 D
B 1278	1272	
Begin	at ⊙ A	and range West.

Content. *37a. 3r. 2p.*

3. Give the plan and area of a field from the following notes.

	A C	
	872	
B 652	731	
	423	545 C
Begin at	⊙ A	and range East.

Area. *5a. 0r. 35p.*

ANOTHER METHOD.

A four-sided field may frequently be surveyed by dividing it into two triangles and a trapezoid, by perpendiculars on its longest side.

TO FIND THE CONTENT.

RULE.—Multiply the sum of the two perpendiculars by their distance on the base line for the double area. The double areas of the two triangles must be found as in Prob. I., and both be added to the double area of the trapezoid; the sum being divided by 2, will give the content required.

EXAMPLES.

1. Required the survey and area of the following field.

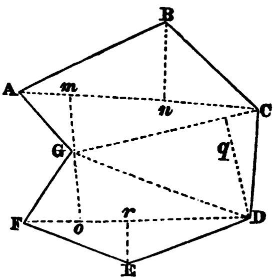
Measure the base A B, and put down in the field book the distances of P and Q, where the perpendiculars rise, &c., as *follows:—*

means of the four conjugate screw *a*, *a*, &c., that move the cross wires, correct for half the error, alternately loosing one screw and tightening its opposite one, till the cross wires cut the same point of the distant object, during an entire revolution of the telescope round its axis.

2. *The adjustment of the bubble tube.*—Move the telescope till it lies in the direction of two of the parallel plate screws, and by giving motion to these screws bring the air-bubble to the centre of its run. Now reverse the telescope carefully in the Ys, that is, change the places of its ends; and should the bubble not settle in the same point of the tube as before, it shows that the bubble tube is out of adjustment, and requires correcting. The end to which the bubble retires must then be noticed, and the bubble made to return one-half the distance by turning the parallel plate screws, and the other half by turning the capstain headed screws at the end of the bubble tube. The telescope must now again be reversed, and the operation repeated, until the bubble settles at the same point of the tube, in the centre of its run, in both positions of the instrument. The adjustment is then perfect, and the clips, that confine the telescope in the Ys should be made fast.

3. *The adjustment of the axis of the telescope perpendicularly to the vertical axis.*—Place the telescope over two of the parallel plate screws, and move them, unscrewing one while screwing up the other, until the bubble of the level settles in the centre of its run; then turn the instrument half round on its vertical axis, so that the contrary ends of the telescope may be over the same two screws, and, if the bubble does not again settle in the same point as before, half the error must be corrected by turning the screw B, and the other half by turning the two parallel plate screws, over which the telescope is placed. Next turn the telescope a quarter round, that it may be over the other two screws, and repeat the same process with these two screws; and when, after a few trials, the bubble maintains the same position in the centre of its run, while the telescope is turned round on the vertical axis, this axis will be truly vertical; and the axis of the telescope being horizontal, by reason of the previous adjustment of the bubble tube, will be perpendicular to the vertical axis, and remain truly horizontal, while the telescope is turned completely round. The adjustment is therefore perfect.

There are several other highly approved levelling instruments, as Troughton's and Gravatt's levels, &c., for the descriptions of which, see *Baker's Land and Engineering Surveying*.



	to ⊙ D	
	520	
p	288	80 E
G 120	206	*o*
	Go to ⊙ F	
	to ⊙ G	
	440	
D 230	152	*q*
	L. of ⊙ C	
	to ⊙ C	
	550	
B 180	410	*n*
m	135	130 G
Begin	at ⊙ A	range E.

CONSTRUCTION.

The above field is divided into two trapeziums A B C G, G D E F and the triangle G C D. Draw the diagonal A C, which make = 550 links; at 135 and 410 set off the perpendiculars *m* C = 130, and *n* B = 180 links respectively; join AB, BC, CG, and GA, and the first trapezium will be completed. Then on CG, lay off C *q* = 152, and draw the perpendicular *q* D = 230; join C D, D G, and the triangle is completed. Lastly, with centre G and radius *o* G = 120 describe an arc; and with centre D and radius *o* D = 314 (= 520−206) describe another arc, intersecting the former in *o*: through *o* draw the diagonal DF = 520 links, upon which, at 288 links, draw the perpendicular *p* E; join D E, E F, F G, and the figure will be completed.

130	440	120	Double areas.
180	230	80	170500 trap. A B C G
——	——	——	101200 tri. C D G
310	13200	200	104000 trap. D E F G
550	880	520	————
———	———	———	2)3·75700
15500	101200	104000	————
1550			1·87850 — 1*a.* 3*r.* 20½*p.*
———			
170500			

2. Required the plan and areas of two fields from the following dimensions.

First Field.

	to ⊙ A	
	504	Base
E 98	233	
Return	to ⊙ B	
	to ⊙ D	
	673	Diag.
C 207	472	
	427	268 B
Begin	at ⊙ A	range W.

Area. 1*a*. 3*r*. 15*p*.

Second Field.

	to ⊙ F	
	970	Diag.
	520	181 B
E 290	413	
	R. of ⊙ D	
	to ⊙ D	
	744	Diag.
C 161	386	
	303	333 B
Begin	at ⊙ A	range W.

Area. 4*a*. 0*r*. 9½*p*.

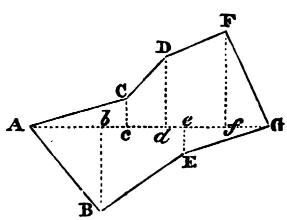
ANOTHER METHOD.

A small piece of land, having several sides, may sometimes be most conveniently measured by taking one diagonal, and upon it erecting perpendiculars to all the angles on each side of it. The piece will thus be divided into right angled triangles and trapezoids, the areas of which must be calculated as in the two last Problems.

EXAMPLES.

1. Required the plan and area of a field from the following notes.

	to ⊙ G	
	1020	
F 470	890	*f*
e	610	50 E
D 320	585	*d*
C 70	440	*c*
b	315	350 B
Begin	at ⊙ A	go E.



NOTE. The method of planning the above field is sufficiently clear, from the *preceding field-notes,* and from what has been already done.

Triangle A C *c*.	Trape. C *c d* D.	Trape. D *d f* F.	Tri. F *f* G.	Tri. A *b* B.
A *c* = 440	D *d* = 320	D *d* = 320	*f* G = 130	A *b* = 315
C *c* = 70	C *c* = 70	F *f* = 470	F *f* = 470	B *b* = 350
30800	sum = 790	sum = 790	9100	15750
	c d = 145	*e f* = 395	52	945
	13050	3950	61100	110250
	435	2370		
	56550	240950		

Trapezoid B *b e* E.	Triangle E *e* G.
B *b* = 350	G *e* = 410
E *e* = 50	E *e* = 50
sum = 400	20500
b e = 295	

Double areas.
30800
56550
240950
61100
110250
118000
20500
2)638150
3·19075

= 3*a*. 1*r*. 38½*p*. Area.

Problem IV.

FIELDS INCLUDED BY ANY NUMBER OF CROOKED OR CURVED SIDES.

When a field or estate is bounded by crooked fences, a line must be measured as near to each of them, as the angles or bends will permit; in doing which an offset must be taken to each corner or bend in the fence. The offsets or perpendiculars thus erected, will divide the whole offset space into right angled triangles and trapezoids, the areas of which may be found as already shewn.

Note 1. When the offsets are short, that is, not greatly exceeding a chain in length, their places on the line may be found by laying the offset-staff at right angles to the chain, as nearly as can be judged by the eye; but when the offsets are large, and correctness is required, their places must be found by the cross, and measured by the chain.

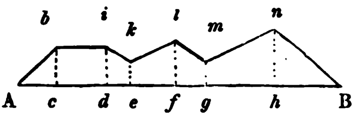
Note 2. The quickest method of laying down offsets, is, by laying the feather-edge of the plotting scale against the base or chain line, and sliding the offset scale along the feather edge to the several distances of the offsets, *and pricking off their* lengths, corresponding to their several distances.

Note 3. Unskilful surveyors usually add all the offsets taken on one line together and divide the sum by their number for a mean breadth; but this method is very erroneous, especially where the offsets vary greatly in length, and should therefore be avoided where great accuracy is required.

EXAMPLES.

1. Required the plan and content of a right-lined piece of ground by offsets, from the following notes.

	to ⊙ B	
o	955	
n 91	785	*h*
m 57	634	*g*
l 88	510	*f*
k 70	340	*e*
i 84	220	*d*
b 62	45	*c*
o	00	
Begin	at ⊙ A	range E.



$Ac = 45$	$cb = 62$	$di = 84$	$ek = 70$	$fl = 88$
$cb = 62$	$di = 84$	$ek = 70$	$fl = 88$	$gm = 57$
90	146	154	158	145
270	$cd = 175$	$de = 120$	$ef = 170$	$fg = 124$
2790	730	18400	11060	580
	1022		158	290
	146		26860	145
	25550			17980

		Double areas.
$gm = 57$	$h'B = 170$	2790
$h'n = 91$	$h'n = 91$	25550
148	170	18480
$gh' = 151$	1530	26860
148	15470	17980
740		22348
148		15470
22348		2) 1·09478
		0·64739 = 0a. 2r. 23p.

5 *

Calculation by the erroneous method (See Note 3).

00	955	= A B.
62	$56\frac{1}{2}$	= mean breadth.
84		
70	5730	
88	4775	
57	477	
91		
00	0·53957	= 0*a*. 2*r*. 6*p*. Content by this method, *which is* 17 *perches too little.* For this method is always erroneous except when the offsets stand at equal distances from one another, and when the first and last offsets are both 0.
8)452		
$56\frac{1}{2}$		

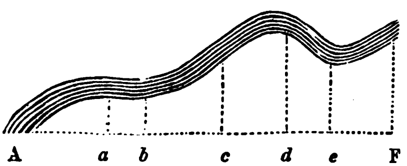
Some omit all the offsets that are 0, dividing the sum of the offsets by the number of real offsets; by this method we shall have

$$6)452 \qquad 75\tfrac{1}{3}$$

$$\begin{array}{r} 955 \\ 75\tfrac{1}{3} \\ \hline 4775 \\ 6685 \\ 318 \\ \hline 0{\cdot}71947 \end{array}$$

0·71947 = 0*a*. 2*r*. 34*p*., *which is* 11 *poles too much.*

2. To lay down a crooked piece of land, adjoining a river from the following notes.



	to ⊙ F	
197	720	
163	600	
226	500	
139	350	
80	200	
74	100	
0	000	[E.
Begin	at ⊙ A	range

The content is found by the same method as in the preceding example.

ectively added to, or subtracted from, the assumed distance the datum line, and the next rise or fall again added to, or ıbtracted from, the sum or difference:—thus 2·15, being a fall, subtracted from 100 (the assumed distance of the datum line) eaving 97·85 feet, the height of the ground at *b*; the next fall 5·75 is then subtracted from 97·85, leaving 91·10 feet for the height at *c*; and so on to 3·53, which is the last fall:—the next 6·57, being a rise, is added, as well as 9·93;—thus the last reduced level is 90·79 feet, which taken from the datum 100 leave 9·21 feet, agreeing with the differences of the sums of the back and fore sights, and of the sums of the rises and falls, and showing the work of casting to be correct. Thus are obtained a series of vertical heights to be set off perpendicularly to the datum line, through the upper extremities of which the sectional line must be drawn.

PRACTICAL LEVEL BOOK.

(Datum line 100 feet below the bench mark at A.)

Back Sights.	Fore Sights.	Rise.	Fall.	Reduced Levels.	Distances.	Remarks.
feet.	feet.	feet.	feet.	feet.	chains.	
				100·00 D		
3·50	5·65		2·15	97·85	4·60	B M on road to lime kilns.
4·10	10·85		6·75	91·10	7·80	
5·04	9·25		4·21	86·89	11·60	
3·84	12·91		9·07	77·82	15·20	Bottom of canal, distant 2·80 chains.
4·12	7·65		3·53	74·29	....	
10·49	3·92	6·57		80·86	21·00	
12·96	3·03	9·93		90·79	27·00	to B M at *g*.
44·05	53·26	16·50	25·71	100·00		
	44·05		16·50			
	9·21	diff. =	9·21	= 9·21		diff. between last reduced level and datum.

In laying down the sectional line from the above columns of reduced levels and distances, the former are always taken from a much larger scale than the latter, otherwise the undulations on the surface of the ground would in many cases be hardly perceptible.

Draw the horizontal line A G, setting off the distances A B, *C, &c., as in the* column of distances, that is A B = 4·6

From the arrangement of the lines in the figure, it is evident that the triangles C A B, C D E are equiangular, and since A C was made = C D, the triangles are equal in all respects, and consequently A B = D E.

NOTE 1. For various other methods of measuring obstructed lines, under different circumstances, see *Baker's Land and Engineering Surveying*, Chap. III.

NOTE 2. A sufficient detail of methods of surveying by the help of the cross, which, though not much used by experienced surveyors, is a simple instrument, and its use readily understood by students. This method is, therefore, a proper introduction to the higher branches of surveying; besides, in rural districts, villages, &c., few surveyors use the more expensive instrument, the chain and cross being found quite sufficient to measure the quantities of growing crops, and other such small surveys as may be there required.

LAND SURVEYING BY THE CHAIN ONLY.

This method of surveying has long been adopted by experienced surveyors, who have found it, in general, more accurate and expeditious, as well as better adapted to laying down extensive surveys especially where no serious obstructions from woodlands, water, buildings, &c., exist; the use of the cross, in this method, being entirely excluded by some surveyors, and by others only used for secondary purposes, as for taking occasionally long offsets, or for squaring of lines obstructed by buildings, water, &c. Instead of the cross some use the optical square for these purposes; while some erect perpendiculars with the chain only. See Chap. III., *Baker's Land and Engineering Surveying*.

PROBLEM I.

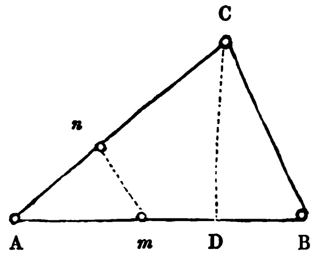
TRIANGULAR FIELDS.

When a triangular field, or piece of ground in that shape, is to be surveyed, set up poles or marks at each corner, and measure each side, leaving marks in at least two of the lines, and entering their distances in the field book; then measure the distance between the two marks for a proof line:—or, one mark only may be left in one of the lines, which may be connected with its opposite angle for a proof line.

EXAMPLES.

1. Required the construction and area of a field from the following dimensions.

Proof From	to ⊙ *n* 384 ⊙ *m*	line.
⊙ *n*	to ⊙ A 1244 700 L ⊙ C	
	to ⊙ C 852 L ⊙ B	
⊙ *m* From	to ⊙ B 1338 1000· 600 ⊙ A	Range E.



When the triangle A B C is constructed, the proof line $m\ n$ will be found to measure 384 links, shewing that there has been no error in the work: also the perpendicular C D will be found to be 770 links; whence the area of the triangle $= 1338 \times 770 \div 2 = 5{\cdot}15130$ $= 5a.\ 0r.\ 24p.$ the area.

Note. If the proof line measured from the plan does not exactly, or very nearly, agree with that measured in the field, some error has been made, and the work must be repeated.

TO FIND THE AREA OF A TRIANGLE FROM THE THREE SIDES.

Rule.—From half the sum of the three sides subtract each side severally and reserve the three remainders; multiply the half sum continually by the three remainders, and the square root of the product will be the area.

Note. By this rule the area of a triangle may be found without laying it down, or finding the perpendicular.

Adopting the preceding example, we have by the rule,

$$\frac{1338 + 852 + 1244}{2} = 1717 = \text{half sum of the three sides.}$$

Then $1717 - 1338 = 379 =$ 1st remainder; $1717 - 852 = 865 =$ 2nd remainder; $1717 - 1244 = 473 =$ 3rd remainder; whence $\sqrt{(1717 \times 379 \times 865 \times 473)} = 5{\cdot}15022 = 5a.\ 0r.\ 24p.$ the sum as the area already found by measuring the perpendicular from the plan.

Note. This method of finding the areas of triangles is very little used in practice, on account of its requiring a tedious calculation, which may, however, be more readily performed by logarithms.

2. It is required to lay down a survey and find its content from the *following field notes.*

The several differences of the sums of the back and fore sights, of the sums of the rises and falls, and of the last reduced level and the datum, exactly agreeing, proves the accuracy of the arithmetical operation in the preceding level book, all these differences being 49·33 feet, which is the height of the last station above the first.

It is advisable for the surveyor to reduce the levels in the field as he proceeds, as it will occupy very little time, and can be easily done while the staffman is taking a new position. The surveyor will thus be enabled to detect with the eye if he is committing any glaring error; for instance, inserting a number in the column of rises, when it ought to be in that of falls, the surface of the ground at once reminding him that he is going downward instead of ascending.

It is seldom the case in practice that the instrument can be placed precisely equi-distant from the back and fore staves, on account of the inequalities of the ground, ponds, &c.; it would appear, therefore, to be necessary, to make our results perfectly correct, to apply to each observation the correction for curvature and refraction as explained at page 118: this, we believe, is seldom done unless in particular cases, where the utmost possible accuracy is required, on account of the smallness of such correction, as may be seen by referring to the table at the end o the book, where this correction for 11 chains is shown to be no more than $\frac{1}{100}$ part of a foot; and as the difference in the distances between the instrument and the fore and back staves can in no case equal that sum, it is evident that such correction may be safely disregarded in practice. Besides, it is not necessary to have the level placed directly between the staves while making observations, as it is frequently inconvenient to do so, for reasons just given, nor does a deviation from a line of the staves, in this respect, in the least affect the accuracy of the result.

The distances in the sixth column of the level book are assumed to be horizontal distances, and in measuring them, care should be taken that they are as nearly such as possible, or they must be afterwards reduced thereto, otherwise the section will be longer than it ought to be. For the purpose of assisting the surveyor in making the necessary reduction from the hypothenusal to the horizontal measure, when laying down the section, a table is given in *Baker's Land and Engineering Surveying*, page 146, shewing the reduction to be made on each chain's length for the several quantities of rise, as shewn by the reading of the staves.

Note. For extensive information on this subject see *Baker's Land and Engineering Surveying*, where an engraved plan and section, adapted to this *example,* are given at the end of the work.

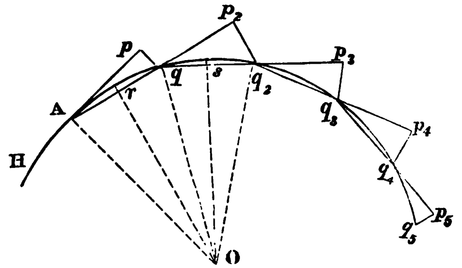
THE METHOD OF LAYING OUT RAILWAY CURVES ON THE GROUND.

In railway practice, the curve adopted is always *an arc of a circle*, to which the straight portions of the railway are tangents at each extremity of the arc. Sometimes the curve consists of two, three, or more circular arcs with their concavities turned in the same or different directions, as in *the compound and serpentine curves.*

PROBLEM I.

To lay out a railway curve on the ground by the common method.

CASE I.—Let H A, q_4 q_5 be the tangental portions of a railway, the extremities A and q_4 of which are required to be united by the circular curve A q_4, to which H A, q_5 q_4 shall be tangents; the radius of the curve being supposed to be previously determined.



Let the radius in this case be 80 chains or one mile; prolong the tangent B A a distance A p = 1 chain; then opposite 80 in table No. 2, at the end of the book, is found 4·95 inches $= p\,q$, which set off at right angles to A p, thus giving the first point in the curve. In the direction A q, measure $q\,p_2 = 1$ chain, and set off $p_2\,q_2 =$ twice $p\,q = 4{\cdot}95 \times 2 = 9{\cdot}9$ inches, at right angles to $q\,p_2$; then q_2 is the second point in the curve. This last operation must be repeated till the curve shall have been set out to the point q_4. Lastly $q_4\,p_5$ being measured = 1 chain, in the direction $q_3\,q_4$, the offset $p_5\,q_5$ will be found = 4·95 inches = the first offset $p\,q$, thus proving the accuracy of the work. In this manner the operation is conducted, whatever be the length of the curve.

CASE II.—Let A O $= r$, and $\delta =$ A $p = q\,p_2$ &c., which may be either less or greater than one chain; then the general length of the first and last offsets $p\,q$, $p_5\,p_5$ is $\frac{\delta^2}{2r}$, and the length of each of the other offsets is $\frac{\delta^2}{r}$, or twice the first or

last offset; but the length of the offsets given in the table is represented by $\frac{1}{2r}$; therefore, if A p, $q\,p_2$, &c., be taken as 2, 3, 4, &c., chains, the value of $\frac{1}{2r}$ must be multiplied by $2^2 = 4$, $3^2 = 9$, $4^2 = 16$, &c., respectively to find $p\,q$, and the result, in each case, multiplied by 2 for each of the offsets $p_2\,q_2$, $p_3\,q_3$, &c. In this manner the curve may be set out more speedily, and with less liability to error, on account of the less number and greater length of the lines required in the operation.

EXAMPLE.

Let A O $= r = 120$ chains, and $\delta = 4$ chains; then $\frac{\delta^2}{2r} = \frac{1}{2r} \times 16 = 3{\cdot}3 \times 16 = 52{\cdot}8$ inches 4 feet 4·8 inches $= p\,q$; whence 4 feet 8 inches $\times\, 2 = 4{\cdot}8$ feet 9·6 inches $= p_2\,q_2, p_3\,q_3 =$ &c.

NOTE 1. When the curve has been correctly set out, as in Case II., the intermediate stumps may be put in at the end of every chain, if required, by the method given in Case I. The distances of the intermediate stumps, thus put in, will not, in most cases, exceed a fraction of an inch; because the lengths of the offsets q, $p_2\,q_2$, &c., is so small, that the curvilinear lengths A q, $q\,q\,q_2$, &c., can never greatly exceed those A p, $q\,p_2$, &c.

NOTE 2. The method given in Case II., is sufficiently accurate when δ does not exceed $\frac{1}{20}$ of the radius of the curve. Besides, at the closing point of the curve, as at q_4, the distance $q_3\,p_4$ is most commonly less or greater than δ. Let $q_3\,p_4 = d$; then the offset $p_4\,q_4$, at the end of the curve is $= \frac{(\delta + d)d}{2r}$; and, when $\delta = 1$ chain, $p^4\,q^4 = \frac{(1 + d)d}{2r}$; or the tabular number for the given radius must be multiplied by $(\delta + d)d$, or by $(1 + d)d$, according as A p, $q\,p_2$, &c., is taken $= \delta$ chains or 1 chain, to give the last offset $p_4\,q_4$; $\frac{1}{2}$ of which is $= p_5\,q_5$, the offset to the tangent $q_4\,q_5$.

EXAMPLE.

Let $r = 120$, and $\delta = 4$ chains, as in the last example, and let $q_3\,p_4 = d = 2{\cdot}68$ chains; then $p_4\,q_4 = \frac{1}{2r} \times (\delta + d)\,d = 3{\cdot}3 \times (4 + 2{\cdot}68) \times 2{\cdot}68 = 59{\cdot}07$ inches; $\frac{1}{2}$ of which, viz., 29·535 inches is $= p_5\,q_5$.

NOTE 3. When δ exceeds $\frac{1}{20}$ of the radius r of the curve, the following formula ought to be used for finding the offsets.

$$p\,q = r\,\sqrt{r^3} - \delta^2,$$

$$\text{and } p^2\,q^2 - \&\text{c.} = \frac{\delta^2}{\sqrt{r^2 - \frac{7}{4}\delta^2}}.$$

See *Baker's Land and Engineering Surveying*, page 164.

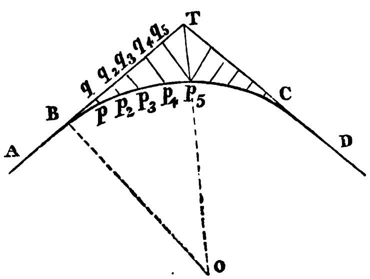
NOTE 4. By this method the greater part of both British and foreign rail-

way curves have been laid out. It was invented by the author about 30 years ago, when the Stockton and Darlington Railway was laid out, and eagerly adopted by engineers as it involves very little calculation, and does not require the use of a theodolite. It is, however, defective in practice, on account of its requiring so very many short lines connected together, as errors will unavoidably creep in and multiply, and more especially so where the ground is rough; thus the curve has frequently to be retraced several times before it can be got right; hence the author prepared the methods in the following Problem.

Problem II.

To lay out a railwag curve on the ground, by offsets from its tangents, no obstructions being supposed to prevent the use of the chain on the convex side of the curve.

Case I.—*When the length of the curve does not exceed $\frac{1}{4}$ of its radius.*



Let A B, D C be straight portions of a railway, the points C and B being required to be joined by a circular curve B C, to which A B, D C shall be tangents, the radius B O of the curve being supposed to be previously determined from an accurate plan of the intended railway.

Range the tangents A B, D C till they meet at T; and let the radius B O = 80 chains = 1 mile; measure on B T the distance B q = 1 chain; and, at right angles to B T, lay off the offset $q p$ = 4·95 inches, by Table No. 2, as in Problem I.; then p is the first point in the curve. Next measure $q q_2$ = 1 chain, and lay off the offset $p_2\ q_2$ = 4·95 × 4 for the second point in the curve. The successive offsets, at the end of every chain, being 4, 9, 16, &c., or 2^2, 3^2, 4^2, &c., times the first offset $p q$, which may also be found opposite the given radius in the Table No. 2., as in Prob. I.

When the offsets have been thus laid out, till the last one $q_5 p_5$ falls little short of T; lay off the same offsets on T C as were laid off in B T, but in an inverted order, making the first distance on T C = Tq_5; thus completing the curve to C.

Note. It can rarely happen in practice that the last offsets, from both tangents, will meet at the middle point p_5 of the short curve, as shewn in the figure; but will either intersect one another or fall short of the middle point; but *this is a matter of* no consequence.

EXAMPLE.

Let the radius of the curve be 160 chains, required the offsets at the end of every chain, from the tangent to the curve.

$p\ q$ per Table (No. 1.) = 2·475 inches.
$p_2 q_2 = 2{\cdot}475 \times 4 = 9{\cdot}9$ ———
$p_3 q_3 = 2{\cdot}475 \times 9 = 22{\cdot}275$ ———
$p_4 q_4 = 2{\cdot}475 \times 16 = 39{\cdot}6$ ———
&c. = &c. = &c.

CASE II.—*To lay out the curve when it is any required length.*

In a long curve (of which there are some more than two miles in length) the tangents, if prolonged to their point of meeting, would necessarily fall at a great distance from the curve, thus giving an inconvenient length to the offsets, which in practice should never exceed two chains. To remedy this inconvenience the curve must be divided into two or more parts, by introducing one or more additional tangents, thus the offsets may be confined within their proper limits. Thus the tangent T C may, in this case, be extended, another tangent applied, and the offsets laid off, thus repeating the operation of Case I. a second time: if the curve be not yet completed, the operation may be repeated a third, fourth, &c., time, till it be completed.

NOTE. For a complete development of this important subject, see *Baker's Land and Engineering Surveing*, Part II., Chap. II., where two other methods of laying out railway curves are given; also methods of laying out compound, serpentine and deviation curves, with original formulæ; all of which methods, as well as the two already given, were first drawn up by the author. See page 179 of the work above referred to, where a short history of the invenion is given. See, also, *Tate's Geometry*, page 247.

CONTENTS OF RAILWAY CUTTINGS, &c.

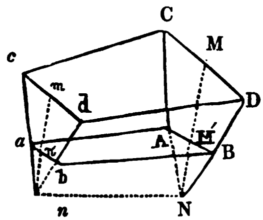
TABLES.

The *General Earthwork Tables*, in conjunction with *Two Auxiliary Tables*, on the same sheet, in *Baker's Railway Engineering*, or the numbers for the slopes in *Bidder's Table*, are applicable to all varieties of ratio of slopes and widths of formation level in common use; and with the help of Barlow's table of square roots, these tables will apply to sectional areas, with all the mathematical accuracy that can be attained, with very little more calculation than adding the contents between every two *cross-sections*, as given by the *General Table*.—The contents in *the General* Table are calculated to the nearest unit, as are also

the Auxiliary Table, No. 2, which is for the decimals of he depths. The Auxiliary Table, No. 1, shews the f the meeting of the side-slopes below the formation- :h the number of cubic yards to be subtracted from the of the General Table for each chain in length, for eight ›st common varieties of ratio of slope.

llowing diagrams and explanations will further illustrate ıod of taking the dimensions of railway cuttings, pre- to using the above named tables.

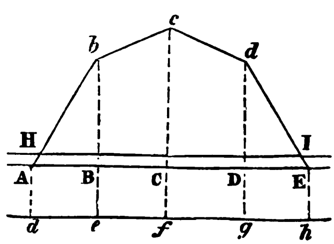
B D C *c a b d*, be a railway of which A B D C, *a b d c* :oss sections, A B = *a b* = 'ormation level, M M′, *m m′* le depths of the two cross- the side-slopes A C, D B, when prolonged two and intersect at N and *n*, at ›ints the prolongations of



, m′ will also meet, thus constituting a prism A B N content of which is to be deducted from the whole given by the General Table, by means of the Table No. hich the depth M′ N = *m n* is also given, as already several varieties of slope and bottom width.

ce this subject in a more practical point of view, let the figure represent a longitudinal and vertical section of a ›assing through the middle A E of the formation level. line of the rails, and *a h*, the line on which the slopes, ıged, would meet.

be seen that the A *b c d* E com- .nd runs out on the ı level A E, and depth A *a* = B *e* : &c., is to be added everal depths B *b*, of the cutting, the last depth at A



ing each = 0; or, what amounts to the same thing, the :pths must be measured from the line *a h*: thus A*a*, *b e*, are the depths to be used. And since the depth A *a* is 'able No. 1, for all the most common cases, or it may be und by calculation for all cases, (see *Railway Engineer-* line corresponding to *a h* must, therefore, be ruled on ay section, at the proper distance below A E, from *e several* depths must be measured; or the vertical

scale may be marked with Indian ink, (which may be readily rubbed off) at the same distance, and this mark may then be applied to the formation level A E, for the purpose of measuring the several depths.—In the case of an embankment, the line for the several depths must be placed at a like distance above the formation level.

Problem I.

The several depths of a railway cutting to the meeting of the side slopes, the width of formation level, and the ratio of the slopes being given, to find the content of the cutting in cubic yards, from the Tables referred to, the distances of the depths being one chain each.

Rule.—Take the several quantities, corresponding to every two succeeding depths of a cutting, or embankment, measured to the meeting of the side slopes, at the distance of 1 chain each, from the General Table in *Baker's Railway Engineering*, and multiply their sum by the ratio of the slopes; from the product subtract the cubic yards, corresponding to the given bottom width and ratio of slopes from Table No. 1., multiplied by the whole length of the cutting, and the remainder will be the content of the cutting in cubic yards.

But, when the distances of the depths are greater or less than 1 chain, the quantities of the General Table must be multiplied by their respective distances.—And, when the distances are given in feet, the quantities must be multiplied by those distances, and the final result divided by 66 for the content in cubic yards, as in the following

EXAMPLES.

1. Let the depth of the railway cutting or embankment to the meeting of the side-slopes, at the end of every chain, be as in the following table, the bottom-width 30 feet, and the ratio of the slopes as 2 to 1; required the content in cubic yards.

Dist, in chains.	Depth in feet.	Qnts. per G. Table.
0	10	
1·00	33	1238
2·00	39	3175
3·00	35	3350
4·00	10	1355
For slope 1 to 1 . .		9128
		2
For slope 2 to 1 .		18256
Subtract 275 × 4	=	1100
Content in cubic yds.	=	17156

Note. In the annexed table the quantity 1238, corresponds to the depths 10 and 33 feet, in the General Table; the quantity 3175 to the depths 33 and 39, and so on for the succeeding depths. By the Auxiliary Table No. 1, it will be seen, that the depth to be added below the formation level, for the given width and ratio of slopes, is 7·50 = $7\frac{1}{2}$ feet, therefore, the cutting begins and ends with a depth of 10 — $7\frac{1}{2}$ = $2\frac{1}{2}$ feet. The corresponding *number of cubic* yards, to be deducted for each chain

h, is multiplied by 4 chains the whole length of the cutting, thus he whole quantity to be deducted, the remainder being the true in cubic yards of the cutting.

The several depths ilway cutting to the g of the side slopes in the annexed the bottom width 30 feet, and the f the slopes $1\frac{1}{2}$ to quired the content cutting.

:. When any of the dis- between two succeed- ths is greater or less chain, the correspond- ntity from the General nust be multiplied by ticular distance; as the es between the depths 25, and between 32 and the distance being 2 The last distance, viz., ween 30 and 10, is 1·46, case 2 figures must be cut off for decimals, after multiplying.

Dist. in chains.	Depths in feet.	Products for Dist. greater than 1 chain.	Total quantities.
0	10		
1·00	16	. . . . .	420
2·00	20	. . . . .	705
4·00	25	1234 × 2	2486
5·00	32	. . . . .	1996
7·00	39	3091 × 2	6182
8·00	45	. . . . .	4319
10·00	50	5520 × 2	11040
12·00	40	4971 × 2	9942
13·00	30	. . . . .	3015
14·46	10	1059 × 1·46	1546
For side slopes 1 to 1			41741
For side slopes $\frac{1}{2}$ to 1			20870
For side slopes $1\frac{1}{2}$ to 1			62611
		366·67 × 14·46 =	5302
		Content in cubic yards	57309

Let the depths of a y cutting to the g of the side slopes, eir distances in feet n the annexed table, ttom width 30 feet, e ratio of the slopes l; required the con- cubic yards.

:. When the distances et the quantities from l Table must be re- ely multiplied by their es, the quantity from No. 1, by the whole dis- nd the result divided the feet in 1 chain, for the content in cubic yards.

Dist. in feet.	Depths in feet.	Quantities multiplied by length.	Total quantities.
0	37		
90	50	4660 × 90	419400
178	61	7554 × 88	664752
278	39	6210 × 100	621000
For slopes 1 to 1			1705152
For slopes $\frac{1}{2}$ to 1			852576
For slopes $1\frac{1}{2}$ to 1			2557728
		366·67 × 278 =	101933
			66)2455795
		Content in cubic yards......	37209

PROBLEM II.

E I.—*The areas of two cross sections of a railway cutting intersection of the side slopes, its length in chains, bottom and the ratio of the slopes are given; required the content cutting in cubic yards.*

LE.—With the square roots of the given areas as depths,

find the content from the General Table, as in the last Problem, from which subtract the quantity answering to the given width, and the ratio of side slopes from Table No. 1, and the remainder, being multiplied by the length, will be the content required.

NOTE. If the length be given in feet, proceed as in Example 3, last Problem.

EXAMPLE.

1. Let the two sectional areas of a cutting be 4761 and 1296 square feet, the bottom width 36 feet, the length 3·25 chains, and the ratio of the side slopes 2 to 1; required the content in cubic yards.

$\left.\begin{array}{l}\sqrt{4761} = 69 \\ \sqrt{1296} = 36\end{array}\right\}$ content per General Table 6959

For bottom width 36 and slopes 2 to 1, per Table No. 1 396

Content for 1 chain in length 6563

3¼

19689

1641

Content for 3·15 chains .... 21330 cubic yds.

CASE II.—*In measuring contract work, where great accuracy is required, the* $\frac{1}{100}$*ths of a foot, or second decimals, must be used in the calculation, by taking for them* $\frac{1}{10}$*th of their respective quantities in Table No. 2.*

EXAMPLE.

The areas of seven cross sections of a railway cutting to the meeting of the side slopes and their distances are as in the annexed table; the bottom width is 30 feet, and the ratio of the slopes 1½ to 1; required the cubic yards in the cutting.

Dist. in chains.	Areas in sq. feet.
0	2727
2·00	3136
6·00	4221
9·00	4100
14·00	5141
16·00	3759
18·00	2161

Ans. The content, per General Table, and Table No. 2, is 172318 cubic yards, from which the quantity corresponding to the given bottom width and ratio of slopes × by the whole length, viz. 275 × 18 = 4950 cubic yards

must be deducted, which leaves 167568 cubic yards, the content required.

NOTE. For further explanations and numerous examples of the methods of finding the contents of earthwork, see *Baker's Land and Engineering Surveying*. See, also, *Tate's Geometry*, page 252.

GENERAL RULE FOR FINDING THE CONTENTS OF SOLIDS.

The wedge, the prismoid, the pyramids, and their frustums; the whole or a segment, or any portion of the whole, contained between two parallel planes perpendicular to the axis of a sphere, of an ellipsoid, of a paraboloid, of an hyperboloid, may be found by the following general formula.

Let A and B be the areas of the ends of the solid, C the area of a section parallel to and equidistant from the ends, and L the distance between the ends; then

$$\text{The solidity} = \frac{A + B + 4\,C}{6} \times L:$$

The following investigation of this very general Rule was given by *B. Gompertz*, Esq., F.R.S., &c., in the *Gentlemen's Mathematical Companion* for 1822.

Let x be the variable distance from a given point of another section parallel to the two ends, and a, b, c be given quantities, the area of the said section will be $a + b\,x + c\,x^2$, as this will contain the cases of the sections of the solids, enumerated in the Rule; for instance, in the pyramid or cone, the area of the section may be expressed by $c\,x^2$, in the wedge the areas may be expressed by $a + b\,x + c\,x^2$, a, b, c being constant for the same point and wedge for any parallel section to a plane given in position, as long as the section has the same number of sides. In the paraboloid all planes perpendicular to the axis, if the given point be the vertex, will have the areas of their sections expressed by $b\,x$. In the ellipsoid or hyperboloid, the point being in that vertex of the axis about which it is revolved, the area of the sections may be expressed by $b\,x + c\,x^2$, c being negative in the ellipsoid and positive in the hyperboloid. And I observe from the method of equidistant ordinates in curves, or of sections in solids, see the method of differences, if A, B represent the areas of the two ends, C the

area of the section in the middle between them, and L the length; then

$$\text{The solidity} = \frac{A + B + 4\,C}{6} \times L \qquad \text{Q. E. I.}$$

EXAMPLES ON THE FOREGOING RULE.

1. The length of a railway cutting is 5 chains or 110 yards, the top width and depth at one end are respectively 120 and 30 feet, the top width and depth at the other end are respectively 90 and 20 feet, and the bottom width 30 feet; required the content of the cutting in cubic yards.
Ans. $20777\frac{7}{9}$ *cubic yards.*

2. Required the content of a sphere, the diameter of which is 12 feet. *Ans.* 33·5104 *cubic yards.*

NOTE. Here the areas of the extreme sections are each $= 0$.

3. A cask is in the form of the middle zone of a sphere, its top and bottom diameters being 34 inches, and its height 30 inches, inside measure; how many gallons will it hold?
Ans. 149·22.

4. How many gallons are contained in a cask, in the form of the middle zone of a spheroid, the bung and head diameter being 40 and 32 inches, and the length 36 inches, all internal measures?
Ans. $143\frac{1}{2}$ *imperial gallons.*

5. How many cubic feet are there in a parabolic conoid, the height of which is 42, and the diameter of its base 24 inches?
Ans. 5·4978 *cubic feet.*

6. A 5 inch cube of ivory is turned into a sphere of the same diameter; what weight of ivory will be lost, its weight being 1820 ounces (Av.) per cubic foot? *Ans.* 61·44 *ounces.*

TABLE No. 1.—THE AREAS OF SEGMENTS OF CIRCLES, DIAMETER UNITY.

Height.	Area Segment.	Height.	Area Segment	Height.	Area Segment.
·001	·000042	·050	.014681	·099	·040276
·002	·000119	·051	·015119	·100	·040875
·003	·000219	·052	·015561	·101	·041476
·004	·000337	·053	·016007	·102	·042080
·005	·000470	·054	·016457	·103	·042687
·006	·000618	·055	·016911	·104	·043296
·007	·000779	·056	·017369	·105	·043908
·008	·000951	·057	·017831	·106	·044522
·009	·001135	·058	·018296	·107	·045139
·010	·001329	·059	·018766	·108	·045759
·011	·001533	·060	·019239	·109	·046381
·012	·001746	·061	·019716	·110	·047005
·013	·001968	·062	·020196	·111	·047632
·014	·002199	·063	·020680	·112	·048262
·015	·002438	·064	·021168	·113	·048894
·016	·002685	·065	·021659	·114	·049528
·017	·002940	·066	·022154	·115	·049165
·018	·003202	·067	·022652	·116	.050804
·019	·003471	·068	·023154	·117	·050446
·020	·003748	·069	·023659	·118	·051090
·021	·004031	·070	·024168	·119	·052736
·022	·004322	·071	·024680	·120	·052385
·023	·004618	·072	·025195	·121	·053036
·024	·004921	·073	·025714	·122	·054689
·025	·005230	·074	·026236	·123	·054345
·026	·005546	·075	·026761	·124	·055003
·027	·005867	·076	·027289	·125	·056663
·028	·006194	·077	·027821	·126	·056326
·029	·006527	·078	·028356	·127	·057991
·030	·006865	·079	·028894	·128	·057658
·031	·007209	·080	·029435	·129	·058327
·032	·007558	·081	·029972	·130	·059999
·033	·007913	·082	·030526	·131	·060672
·034	·008273	·083	·031076	·132	·061348
·035	·008698	·084	·031629	·133	·062026
·036	·009008	·085	·032186	·134	·062707
·037	·009383	·086	·032745	·135	·063389
·038	·009763	·087	·033307	·136	·064074
·039	·010148	·088	·033872	·137	·064760
·040	·010537	·089	·034441	·138	·065449
·041	·010931	·090	·035011	·139	·066140
·042	·011330	·091	·035585	·140	·066833
·043	·011734	·092	·036162	·141	·067528
·044	·012142	·093	·036741	·142	·068225
·045	·012554	·094	·037323	·143	·068924
·046	·012971	·095	·037909	·144	·069625
·047	·013392	·096	·038496	·145	·070328
·048	·013818	·097	·039087	·146	·071033
·049	·014247	·098	·039680	·147	·071741

NOTE. When the tabular height exceeds the heights given in this Table he segment must be divided into two equal parts and their common versed ine found by Prob. VI., page 26. The tabular heights will then fall within *his Table,* whence the area of the whole segment may be found.

No. 2.—Offsets at the end of the first chain from tangent point of railway curves.

Radius of curve in chns.	Offsets in inches and decimals.	Radius of curve in chns.	Offsets in inches and decimals.	Radius of curve in chns.	Offsets in inches and decimals.	Radius of curve in chns.	Offsets in inches and decimals.
40	9·9000	64	6·1875	88	4·5000	120	3·3000
41	9·6588	65	9·0923	89	4·4496	122	3·2459
42	9·4285	66	6·0000	90	4·4000	124	3·1935
43	9·2093	67	5·9104	91	4·3516	125	3·1680
44	9·0000	68	5·8235	92	4·3043	126	3·1428
45	8·8000	69	5·7391	93	4·2581	128	3·0937
46	8·6087	70	5·6571	94	4·2128	130	3·0461
47	8·4255	71	5·5774	95	4·1684	132	3·0000
48	8·2500	72	5·5000	96	4·1250	134	2·9552
49	8·0816	73	5·4246	97	4·0825	135	2·9333
50	7·9200	74	5·3513	98	4·0408	136	2·9117
51	7·7647	75	5·2800	99	4·0000	138	2·8645
52	7·6154	76	5·2105	100	3·9600	140	2·8285
53	7·4717	77	5·1428	102	3·8824	142	2·7887
54	7·3333	78	5·0769	104	3·8077	144	2·7500
55	7·2000	79	5·0126	105	3·7714	145	2·7310
56	7·0714	80	4·9500	106	3·7358	146	2·7123
57	6·9473	81	4·8889	108	3·6667	148	2·6756
58	6·8276	82	4·8292	110	3·6000	150	2·6400
59	6·7118	83	4·7711	112	3·5352	152	2·6052
60	6·6000	84	4·7143	114	3·4736	154	2·5714
61	6·4918	85	4·6588	115	3·4435	155	2·5548
62	6·3871	86	4·6046	116	3·4138	156	2·5384
63	6·2857	87	4·5517	118	3·3599	158	2·5063

TABLE OF CORRECTIONS FOR CURVATURE, ETC.

No. 3.—Difference between apparent and true level for distances in chains. Correction in decimals of feet.

Distance in chains.	For curvature and refraction.	Distance in chains.	For curvature and refraction.
3½	·001	14	·017
4	·002	14½	·019
4½	·002	15	·020
5	·003	15½	·021
5½	·003	16	·023
6	·003	16½	·024
6½	·004	17	·026
7	·004	17½	·027
7½	·005	18	·029
8	·006	18½	·031
8½	·006	19	·033
9	·007	19½	·034
9½	·008	20	·036
10	·009	20½	·038
10½	·009	21	·039
11	·011	21½	·041
11½	·012	22	·043
12	·013	22½	·046
12½	·014	23	·047
13	·016	23½	·049
13½	·016	24	·051

No. 4.—Difference between apparent and true level for distances in miles. Correction in feet and decimals.

Distance in miles.	For curvature and refraction.	Distance in miles.	For curvature and refraction
¼	·03	10	57·17
½	·15	10½	61·30
¾	·32	11	69·16
1	·58	11½	75·59
1½	1·29	12	82·29
2	2·29	12½	89·29
2½	3·57	13	96·58
3	5·14	13½	104·14
3½	7·00	14	112·00
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